

ELECTRODYNAMICS  
PROBLEM SET 2  
due February 9, before class

**Problem 1.: Field invariants**

- a) Write the Lorentz scalars  $F_{\mu\nu}F^{\mu\nu}$  and  $\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}$  in terms of  $\vec{E}$  and  $\vec{B}$ .
- b) Show that if  $\vec{E}$  and  $\vec{B}$  are seen as perpendicular in one reference frame, they are perpendicular in any other reference frame.
- c) Show that if  $|\vec{E}| = |\vec{B}|$  in one reference frame,  $|\vec{E}'| = |\vec{B}'|$  in any other reference frame.
- d) Show that if  $|\vec{E}| > |\vec{B}|$  in one reference frame,  $|\vec{E}'| > |\vec{B}'|$  in any other reference frame.

**Problem 2.: Relativistic invariance of Maxwell's equations in your face**

An infinitely long straight wire of negligible cross sectional area is at rest and has a uniform charge density  $q_0$  in the inertial frame  $K'$ . The frame  $K'$  (and the wire) move with velocity  $v$  parallel to the direction of the wire with respect to the lab frame  $K$ .

- a) Write down the electric and magnetic fields in cylindrical coordinates in the rest frame of the wire. Using the Lorentz transformation properties of the fields, find the components of the electric and magnetic fields in the lab.
- b) What are the charge and current densities associated with the wire in its rest frame? In the lab frame?
- c) From the lab frame charge and current densities, calculate directly the electric and magnetic fields in the lab frame. Compare with part a).

**Problem 3.: Field transformations**

In a certain reference frame a static, uniform, electric field  $E_0$  is parallel to the  $x$  axis, and a static, uniform, magnetic field  $B_0 = 2E_0$  lies in the  $x - y$  plane, making an angle  $\theta$  with the  $x$  axis. Determine the relative velocity of a reference frame in which the electric field and magnetic fields are parallel. What are the fields in that frame for  $\theta \ll 1$  and  $\theta \rightarrow \pi/2$ ?

**Problem 4.: Cyclotron frequency**

A particle with mass  $m$  and charge  $q$  moves in a constant, homogeneous magnetic field. Suppose the initial velocity is perpendicular to the field. What will be the trajectory of the particle? Let us put in some numbers now. Imagine that the particle is an electron and the magnetic field is among the largest routinely created (10 Tesla, in MRI machines). What would be the radius of the trajectory of the energy of the electron is 1) 1000 eV (like in a TV tube) 2) 100 GeV (as in a particle accelerator. Hint: I'm using units that are convenient to me; most of the work here is in the conversion of units.

**Problem 5.: Isolated electrons do not radiate**

Show that a charge particle with non-vanishing mass cannot emit a photon (with zero mass) and stay with the same mass.

**Problem 6.: Colliders**

Suppose a particle of mass  $m$  and energy  $E$  collides with another particle of mass  $m$  initially at rest. Experiments of this type are called "fixed target" because, well, the target is fixed. If  $E$  is large enough, particles with high mass can be created. However, conservation of 3-momentum implies that most of the energy has to be transformed into kinetic energy and only a fraction can be used for the formation of new particles. In order to find the amount of energy available for the formation of new particles we can consider the center-of-mass frame, that is, the frame in which the total 3-momentum vanishes. The energy in that frame is the amount of energy that can be transformed into rest mass. What is the total energy in the center of mass frame and a function of  $E$  and  $m$ ?

If you did the previous problem correctly you should find that for  $E \gg m$  the energy in the center-of-mass frame is a small fraction of  $E$ . If you want to make new particles a better strategy is to spend twice the money and build two accelerators, each one with energy  $E$ , and make head on collisions. That's called a "collider". What is the energy in the center of mass system now? Is this energy more than twice the energy in the "fixed target" experiment?