HW #4 Phys 606

Problem 1

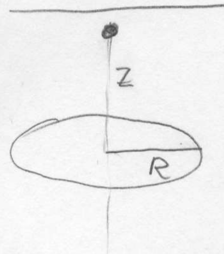
$$\Phi_{avg} = \frac{1}{4\pi R^2}\int \sin\theta \, d\phi \, d\theta \cdot R^2 \frac{q}{\sqrt{R^2+r^2-2Rr\cos\theta}}$$

$$= \frac{-1}{4\pi}\int_0^{2\pi} d\phi \int_1^{-1} d(\cos\theta) \frac{q}{\sqrt{R^2+r^2-2Rr\cos\theta}} =$$

$$= \frac{1}{2} q \cdot \frac{-1}{-2Rr}\left[\sqrt{R^2+r^2-2Rr\cos\theta}\right]_{+1}^{-1} 2 =$$

$$= \frac{1}{2} q \frac{1}{2Rr}\left(R+r-r+R\right) = \frac{q}{r}$$

Problem 2



a) $\Phi(z) = \dfrac{2\pi R\lambda}{\sqrt{R^2+z^2}}$

b) we know that the general solution should be of the form

$$V(r,\theta) = \sum_\ell \left(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}}\right) P_\ell(\cos\theta)$$

On the other hand on z axis for $r>R$

$$\phi(r, \theta=0) = \frac{2\pi R\lambda}{\sqrt{R^2+r^2}} \quad \text{if } r>R$$

$$\frac{2\pi R\lambda}{r\sqrt{1+\frac{R^2}{r^2}}} = \frac{2\pi R\lambda}{r}\left[1-\frac{1}{2}\left(\frac{R^2}{r^2}\right)+\frac{3}{8}\left(\frac{R^2}{r^2}\right)^2-\frac{5}{16}\left(\frac{R^2}{R^2}\right)^3+\dots\right]$$

$\Rightarrow A_\ell = 0$, using $P_\ell(-1) = P_\ell(1) = 1$

$B_0 = 2\pi R\lambda$ $\qquad B_2 = -\pi R^3\lambda$

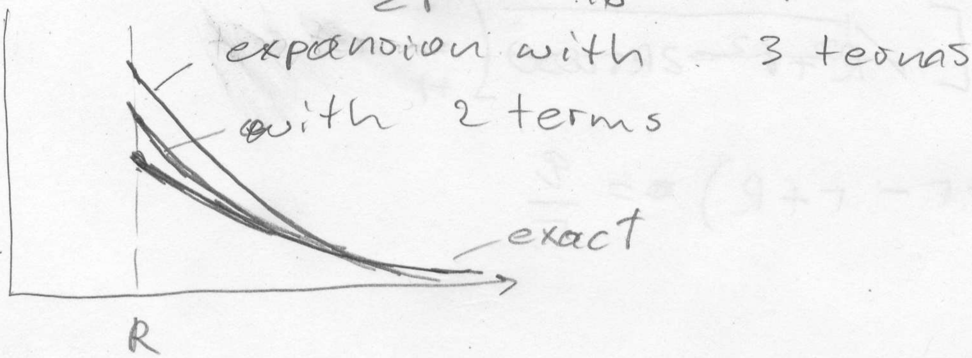
$B_1 = 0$ $\qquad B_3 = 0$ $\qquad B_4 = \frac{3}{4}\pi R^5;$

On the axis of the ring

$$\Phi_{exact} = \frac{2\pi R \lambda}{r'}$$

$$\Phi_{appr} = \frac{2\pi R\lambda}{r}\left[- \frac{\pi R^3 \lambda}{r^3} P_2(0) + \frac{3}{4}\frac{\pi R^5 \lambda}{r^5} P_4(0)\right)$$

$$= \frac{2\pi R\lambda}{r} + \frac{\pi R^3 \lambda}{2r^3} + \frac{9}{16}\pi\lambda\frac{R^5}{r^5} + \dots$$



Problem 3

$$D_{\alpha\beta} = \sum e_i \left(3x_\alpha x_\beta - r^2 \delta_{\alpha\beta}\right)$$

So we have to compute integrals

$$\int dx\, dy\, \sigma \cdot x_i x_j$$

1) all integrals with z vanish

2) all integrals with single power of x or y vanish from symmetry $x \to -x$

$\Rightarrow$ we have only one nonvanishing integral

$$\int dx\, dy\, \sigma \cdot x^2 = \int dx\, dy\, \sigma \cdot y^2 = \frac{1}{2}\int dx\, dy\, \sigma\,(x^2+y^2) =$$

$$= \frac{1}{2}\int_0^R 2\pi r\, dr\, \sigma \cdot r^2 = \frac{\sigma\pi R^4}{4}$$

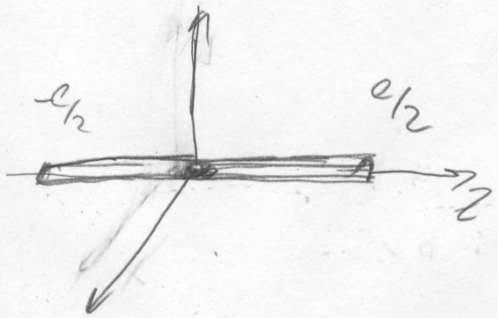
$$D_{xy} = D_{yz} = D_{xz} = 0$$

1

~~Dx~~

$$D_{xx} = \frac{\sigma \pi R^4}{4} \qquad D_{zz} = -\frac{\sigma \pi R^4}{2}$$

$$D_{yy} = \frac{\sigma \pi R^4}{4}$$



Only nonvanishing integral

$$\int_{-\ell/2}^{\ell/2} dz \cdot \lambda \cdot z^2 = \frac{\lambda}{12} \ell^3$$

$$D_{zz} = \frac{\lambda}{6} \ell^3 \qquad D_{yy} = -\frac{\lambda}{12} \ell^3$$

$$D_{xx} = -\frac{\lambda}{12} \ell^3$$

$$D_{xy} = D_{yz} = D_{xz} = 0$$