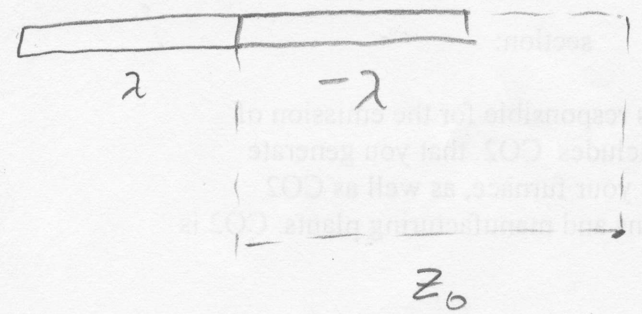
~~PP~~ Phys 606 HW #3



a) $$\Phi(y_0,z_0) = \lambda \int_{-L/2}^{0} \frac{dz}{\sqrt{y^2+(z_0-z)^2}} - \lambda \int_{0}^{L/2} \frac{dz}{\sqrt{y^2+(z_0-z)^2}} =$$

$$= -\lambda \left[\ln\left| \frac{z_0+\sqrt{y^2+z_0^2}}{z_0+L/2+\sqrt{(z_0+L/2)^2+y^2}} \right| - \ln\left| \frac{z_0-L/2+\sqrt{(z_0-L/2)^2+y^2}}{z_0+\sqrt{y^2+z_0^2}} \right| \right] =$$

$$= -\lambda \ln\left[\frac{(z_0+\sqrt{y^2+z_0^2})^2}{(z_0+L/2+\sqrt{(z_0+L/2)^2+y^2})(z_0-L/2+\sqrt{(z_0-L/2)^2+y^2})} \right] \quad (*)$$

b) Total charge

$$Q = \int_{-L/2}^{0} \lambda\, dz - \int_{0}^{L/2} \lambda\, dz = 0$$

$$\vec{d} = \int_{-L/2}^{0} \lambda z\, dz - \int_{0}^{L/2} \lambda z\, dz = -\frac{\lambda L^2}{4}$$

$$D_{zz} = \int_{-L/2}^{0} \lambda (3z^2 - z^2)\, dz - \lambda \int_{0}^{L/2} \lambda (3z^2 - z^2)\, dz = 0.$$

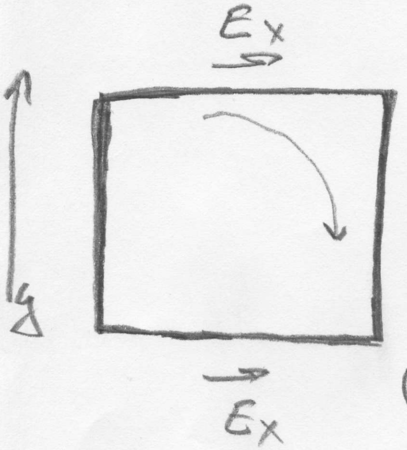
$D_{yy} = D_{yz} = 0$ similarly for $x \,\&\, y \Rightarrow$

$$\phi = \frac{\vec{d}\,\vec{R}_0}{R_0^3} = -\frac{\lambda L^2}{4} \frac{z_0}{R_0^3} \qquad R_0 = \sqrt{z_0^2+y_0^2}$$

Expanding (*) for $L << R$ we will get

$$\Phi = -\lambda \ln\left[\frac{(z_0+R_0)^2}{(z+R)^2\left(1-\frac{L^2 z_0^2}{4R_0^3}\right)}\right] = -\lambda \frac{L^2}{4}\frac{z}{R_0^3}$$

Problem 2



$$I = \frac{n_+ q v_+}{a} = \frac{n_- q v_-}{a}$$

a) $$P = \frac{m n_+ v_+}{\sqrt{1-v_+^2/c^2}} - \frac{m n_- v_-}{\sqrt{1-v_-^2/c^2}}$$

in the limit $v \to 0$

$$P \simeq m n_+ v_+ - m n_- v_- = m\left(\frac{Ia}{q} - \frac{Ia}{q}\right) = 0$$

$$P = \frac{m n_+ v_+}{\sqrt{1-v_+^2/c^2}} - \frac{m n_- v_-}{\sqrt{1-v_-^2/c^2}} = \frac{maI}{q}\left(\frac{1}{\sqrt{1-v_+^2/c^2}} - \frac{1}{\sqrt{1-v_-^2/c^2}}\right) =$$

$$= \frac{maI}{qc^2}\left(\frac{mc^2}{\sqrt{1-v_+^2/c^2}} - \frac{mc^2}{\sqrt{1-v_-^2/c^2}}\right) = \frac{aI}{qc^2}\underbrace{(E_+ - E_-)}_{\text{change of kinetic energy}}$$

but change of energy is work done by electric force $(E_+ - E_-) = q E_x \cdot a \Rightarrow$

$$P = \frac{aI}{qc^2} q E_x a = \frac{E_x I a^2}{c^2} = \frac{E_x m}{c}$$

$m = \frac{Ia^2}{c}$ - magnetic moment of the loop of wire.