

ELECTRODYNAMICS
 PROBLEM SET 3
due March 3rd, before class

Problem 1.: Multipole expansion

Consider a linear distribution of charge along the z-axis where the charge density from $z = 0$ to $z = L/2$ is λ and from $z = 0$ to $z = -L/2$ is $-\lambda$.

- a) Calculate the potential by direct integration.
- b) Calculate the total charge, dipole moment and quadrupole moment of the charge distribution.
- c) Calculate the potential using the multipole expansion up to third (quadrupole) order.
- d) Compare the results in a) and c) and verify that they agree to the order they should agree.

Problem 2.: Hidden momentum

In the discussion of the energy flow in a coaxial cable in class we argue that the Poynting vector was non-vanishing inside the cable and pointed from the battery to the lamp. Since the momentum of the electromagnetic field is proportional to the Poynting vector, this result implies that the momentum of the electromagnetic fields is not zero. Since the total momentum of the charges in motion seem to be zero, this result would imply that the total momentum of the system is zero. For a static system, this is very odd ...

It turns out that the total mechanical momentum of the charges is not zero. In fact, it exactly cancels the electromagnetic one so the total momentum of the system is zero after all. The origin of this “hidden momentum” is a subtle relativistic effect so, instead of calculating it for the coaxial cable, we will use a simpler geometry.

Consider a square wire loop lying on the xy plane carrying a current I and immersed in an external constant electric field in the direction \hat{x} . The current generates a magnetic field that, combined with the external electric field, induces an electromagnetic momentum. In this problem we will show that there is a mechanical momentum in the currents going around the wire loop that cancels that.

a) Imagine that the current is carried by N_+ charges moving at velocity v_+ in one loop segment along \hat{y} and N_- charges with velocity v_- along the other segment along \hat{y} . $v_+ \neq v_-$ because the charges are accelerated (decelerated) in the segments along \hat{x} . The current is

$$I = \frac{N_+ q v_+}{a} = \frac{N_- q v_-}{a}, \quad (1)$$

where q is the charge of each particle and a the size of the loop. What is the total mechanical (relativistic) momentum of the charges in the loop?

- b) What would the the momentum if we used non-relativistic mechanics instead?
- c) Write the mechanical momentum of the charges in terms of the magnetic dipole of the loop and the external electric field.