ELECTRODYNAMICS

PROBLEM SET 1

due February 8, before class

Problem 1.: Most of vector analysis The components notation ("indices galore") and tensors are also useful in 3 dimensions. The trick is to write the vector product with the help of the 3-D ϵ^{ijk} tensor as

$$(\vec{A} \times \vec{B})^i = \epsilon^{ijk} A^j B^k. \tag{1}$$

a) Show that $\epsilon^{ijk}\epsilon^{ilm} = \delta^{jl}\delta^{km} - \delta^{jm}\delta^{kl}$ (hint: what else could it be?)
b) Show that $\epsilon^{ijk}\epsilon^{ijm} = 2\delta^{km}$.
c) Show that

$$\begin{aligned}
\vec{A}.(\vec{B} \times \vec{C}) &= \vec{B}.(\vec{C} \times \vec{A}) = \vec{C}.(\vec{A} \times \vec{B}) && (2)\\
\nabla(\vec{A}.\vec{B}) &= \vec{A} \times (\nabla \times \vec{B}) + B \times (\nabla \times \vec{A}) + (\vec{A}.\nabla)\vec{B} + (\vec{B}.\nabla)\vec{A} && (3)\\
\nabla.(\vec{A} \times \vec{B}) &= \vec{B}.\nabla \times \vec{A} - \vec{A}.\nabla \times \vec{B} && (4)\\
\nabla.\nabla \times \vec{A} &= 0 && (5)\\
\nabla \times \nabla f &= 0 && (6)\\
\nabla \times (\nabla \times \vec{A}) &= \nabla(\nabla.\vec{A}) - \nabla^2 \vec{A}, && (7)
\end{aligned}$$

where $\vec{A}, \vec{B}$ and $\vec{C}$ are vector fields and f a scalar field (all well behaved enough). Since we are talking about 3-vectors only, it doesn't pay to distinguish between covariant and contravariant components in this problem, so take $A_i = A^i$ and $\epsilon^{123} = \epsilon_{123} = 1$.

Problem 2.: Field invariants

a) Write the Lorentz scalars $F_{\mu\nu}F^{\mu\nu}$ and $\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}$ in terms of $\vec{E}$ and $\vec{B}$.

b)Show that if $\vec{E}$ and $\vec{B}$ are seen as perpendicular in one reference frame, they are perpendicular in any other reference frame.

c) Show that if $|\vec{E}| = |\vec{B}|$ in one reference frame, $|\vec{E'}| = |\vec{B'}|$ in any other reference frame.

d) Show that if $|\vec{E}| > |\vec{B}|$ in one reference frame, $|\vec{E'}| > |\vec{B'}|$ in any other reference frame.

Problem 3.: Field of moving charge

What are the $\vec{E}$ and $\vec{B}$ fields of a point particle with charge q moving along the z axis with constant speed v. Sketch the fields.

Problem 4.: tensor (anti)-symmetry

Show that if $A_{\mu\nu} = A_{\nu\mu}$ is a symmetric tensor in one frame, it'll be symmetric in any other frame. Is *anti*-symmetry ($A_{\mu\nu} = -A_{\nu\mu}$) also frame independent?

Problem 5.: Relativistic velocity composition

Multiply two Lorentz transformation matrices with velocities v_1 and v_2 (you can take v_1 parallel to v_2) and show that the result also looks like a Lorentz transformation with velocity v_3. What is v_3 as a function of v_1 and v_2?

Problem 6.: Relativistic invariance of Maxwell's equations in your face

An infinitely long straight wire of negligible cross sectional area is at rest ans has an uniform charge density q_0 in the in the inertial frame K'. The frame K' (and the wire) move with velocity v parallel to the direction of the wire with respect to the lab frame K.

a) Write down the electric and magnetic fields in cylindrical coordinates in the rest frame of the wire. Using the Lorentz transformation properties of the fields, find the components of the electric and magnetic fields in the lab.

b) What are the charges and current densities associated with the wire in its rest frame? In the lab frame?

c) From the lab frame charge and current densities, calculate directly the electric and magnetic fields in the lab frame. Compare with part a).