ELECTRODYNAMICS
PROBLEM SET 8
due April 15^{th} *, before class*

I. DRAWING WITH CRAYONS

Compute the TE and TM modes in a rectangular cavity. Then, to the best of your abilities, make a sketch of the field lines in both cases. You are encouraged to use computer graphics, ray tracing, ... The best one gets posted on the course webpage.

II. IF THE QUALIFY WERE THIS WEEK, WOULD YOU PASS IT?

An electromagnetic wave with electric field

$$\vec{E} = \Re\left(E_y(x,z)e^{-i\omega t}\hat{y}\right) \tag{1}$$

propagates through a medium with dielectric constant $\epsilon(x)$ and permeability $\mu = 1$.

a) Derive a differential equation (from Maxwell's equations) for the complex amplitude $E_y(x,z)$.

b) Obtain expressions for the magnetic field and verify that the Maxwell's equations are satisfied.

c) Show that solutions that are nearly plane waves ($E_y(x,z) = \tilde{E}_y(x,z)e^{ik_0 z}$ with $k_0\tilde{E}_y(x,z) \gg \partial\tilde{E}_y(x,z)/\partial z$) satisfy an equation of the form,

$$2ik_0\frac{\partial}{\partial z}\tilde{E}_y(x,z) + \frac{\partial^2}{\partial^2 x}\tilde{E}_y(x,z) - V(x)\tilde{E}_y(x,z) = 0, \tag{2}$$

which is analogous to the Schrödinger equation. What plays the role of the potential $V(x)$?

d) If the medium is homogeneous (ϵ = constant) a gaussian shaped wave spread in x as it propagates in z.

$$\tilde{E}_y(x,z) = \frac{E_0}{\sqrt{\pi\sigma(z)}}e^{-\frac{x^2}{\sigma(z)}}. \tag{3}$$

Obtain an expression for the z-dependent complex width σ in terms of the initial value $\sigma(0)$ and the wave number k_0 for the special value of k_0 such that $V = 0$.

e)How must the dielectric constant depend on x and what must be the wave number k_0 be in order that the ewave does not spread as it propagates?

No creative problem this week !