ELECTRODYNAMICS
PROBLEM SET 4
due March 9, before class

I. ENERGY, FORCES AND TORQUES ON MULTIPOLES

Consider static charge distribution characterized by a charge q, an electric dipole $\vec{p}$ and an electric quadrupole **Q** in an *external* electric field. Assuming that the external field varies slowly within the size of the charge distribution:

a) Show that the energy of the charge distribution due to the interaction with the external field is given by

$$U = q\phi(0) + \#\vec{p}.\vec{E} + \#Q_{ij}\partial_i E_j + \cdots , \tag{1}$$

where # are numerical coefficients. Determine the #'s.

b) Using the result above, calculate the energy of one electric dipole in the electric field of another dipole (assuming their separation is much larger than their sizes).

c) What s the analogue expression for the force acting on the charge distribution?

Consider now a localized steady current distribution immersed on an *external* magnetic field varying little within the size of the current distribution.

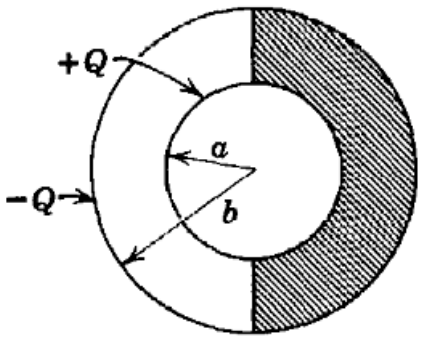
d) What is the expression for the energy of a magnetic dipole?

e) What is the force exerted by the external field on the dipole?

f) What is the torque exerted on the dipole?

II. ARTIFICIAL PROBLEM WITH DIELECTRICS AND BOUNDARY VALUES

Two conduction spherical shells of radii a and b carry charge Q and $-Q$. The empty space between them is half-filled by a dielectric with dielectric constant ϵ. Find the electric field between the spheres.



III. CREATIVITY

Choose/make-up a problem and solve it.