ELECTRODYNAMICS

PROBLEM SET 3

due September 25, before class

I. CYLINDER BOUNDARY PROBLEM

Two halves of a long, hollow cylinder of radius b are separated by small, lengthwise gaps, are kept at potentials V_1 and V_2. Show that the potential inside the cylinder can be written as

$$\phi = \frac{V_1 + V_2}{2} + \frac{V_1 - V_2}{\pi} \mathrm{arctg}\left(\frac{2br\sin\theta}{b^2 - r^2}\right). \tag{1}$$

Hint:

$$\sum_{nodd} \frac{z^n}{n} = \tfrac{1}{2}\log\left(\frac{1+z}{1-z}\right), \tag{2}$$

and $\mathbb{I}m \log z = \mathrm{phase}(z)$.

II. MULTIPOLE FIELDS

a) Calculate the electric field of an electric dipole $\vec{P}$. *Warning:* there is a delta function potential contribution at the origin, don't miss it. Sketch the field lines.

b) Calculate the electric field of an electric dipole Q^{ij} given by:

$$\mathbf{Q} = \begin{pmatrix} Q & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & -2Q \end{pmatrix}. \tag{3}$$

Sketch the field lines.

III. GREEN'S FUNCTION FOR THE DIRICHLET PROBLEM ON A CYLINDER

A point charge q is located at the point (ρ', φ', z') inside a grounded cylindrical box defined by the surfaces $z = 0, z = L$ and $\rho = a$. Find the potential inside the cylinder. Depending on the method you use you'll get one of the three forms of the solution found in problem 3.23 in Jackson's book. Can you show the equivalence between them or use different methods to derive more than one form?

IV. CREATIVITY

a) Make up a problem.

b) Solve it.