

ELECTRODYNAMICS  
PROBLEM SET 1  
due February 9, before class

**Problem 1.: Field invariants**

- a) Write the Lorentz scalars  $F_{\mu\nu}F^{\mu\nu}$  and  $\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}$  in terms of  $\vec{E}$  and  $\vec{B}$ .
- b) Show that if  $\vec{E}$  and  $\vec{B}$  are seen as perpendicular in one reference frame, they are perpendicular in any other reference frame.
- c) Show that if  $|\vec{E}| = |\vec{B}|$  in one reference frame,  $|\vec{E}'| = |\vec{B}'|$  in any other reference frame.
- d) Show that if  $|\vec{E}| > |\vec{B}|$  in one reference frame,  $|\vec{E}'| > |\vec{B}'|$  in any other reference frame.

**Problem 2.: Field of moving charge**

What are the  $\vec{E}$  and  $\vec{B}$  fields of a point particle with charge  $q$  moving along the  $z$  axis with constant speed  $v$ . Sketch the fields.

**Problem 3.: tensor (anti)-symmetry**

Show that if  $A_{\mu\nu} = A_{\nu\mu}$  is a symmetric tensor in one frame, it'll be symmetric in any other frame. Is *anti*-symmetry ( $A_{\mu\nu} = -A_{\nu\mu}$ ) also frame independent?

**Problem 4.: Relativistic velocity composition**

Multiply two Lorentz transformation matrices with velocities  $v_1$  and  $v_2$  (you can take  $v_1$  parallel to  $v_2$ ) and show that the result also looks like a Lorentz transformation with velocity  $v_3$ . What is  $v_3$  as a function of  $v_1$  and  $v_2$ ?

**Problem 5.: Relativistic invariance of Maxwell's equations on your face**

An infinitely long straight wire of negligible cross sectional area is at rest and has a uniform charge density  $q_0$  in the inertial frame  $K'$ . The frame  $K'$  (and the wire) move with velocity  $v$  parallel to the direction of the wire with respect to the lab frame  $K$ .

- a) Write down the electric and magnetic fields in cylindrical coordinates in the rest frame of the wire. Using the Lorentz transformation properties of the fields, find the components of the electric and magnetic fields in the lab.
- b) What are the charges and current densities associated with the wire in its rest frame? In the lab frame?
- c) From the lab frame charge and current densities, calculate directly the electric and magnetic fields in the lab frame. Compare with part a).