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Physics 603 **HOMEWORK ASSIGNMENT #5** Spring 2013

Due date for problems on Thursday, April 4 [deadline on April 9].

1. PB 5.1

2. Statistics of deuterium:

a) Discuss the wave functions needed to describe molecules of ortho-deuterium (o-D_2) and para-deuterium (p-D_2).

b) Write down the partition functions for a system of N molecules of i) o-D_2, ii) p-D_2, iii) equilibrium D_2. Note that it is not straightforward to write the partition function for normal D_2,

c) Find expressions for the equilibrium ratios of o-D_2 to p-D_2 at i) very high T and ii) low T.

3. PB 6.1 and the following part of 6.2:

For BE and FD statistics show that $\langle n_\varepsilon^2\rangle - \langle n_\varepsilon\rangle^2 = \langle n_\varepsilon\rangle \pm \langle n_\varepsilon\rangle^2$, respectively. Show then that in both cases the right-hand side is $k_B T\left(\frac{\partial\langle n\rangle}{\partial\mu}\right)_T$.

4. (5) PB 6.10. Part a should be familiar and is mostly to help you do part b. In part b, relate d T/T to dp/p for adiabatic systems and then use your result in part a.

5. Old qualifier problem, on next page.

1.3 In honor of the Gibbs sesquicentenial:

Consider a total number of N noble gas molecules (monatomic) confined in a cubical box of volume V, area A, and length L. The density of the gas can be described by figure 1.

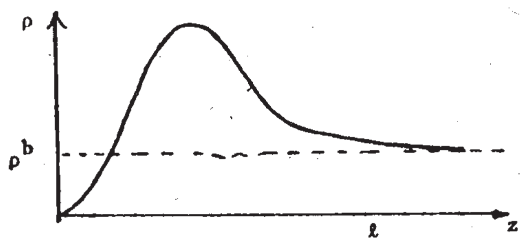

Fig. 1

The gas density $\rho(z)$ differs from the bulk density ρ^b only over a distance $\ell \ll L$ from a wall.

1. What is the physical origin of the peak in ρ? On figure 1, indicate roughly an estimate of the radius R of a gas atom.

2. One can divide the box into bulk and surface regions, as depicted in figure 2, with $V = V^b + V^s$ and V^s arbitrarily defined, similarly $N = N^b + N^s$. By definition $N^b = \rho^b V^b$. Show that if the dotted line is moved, $\Delta N^s = \rho^b \Delta V^s$, i.e. independent of the details of figure 1.

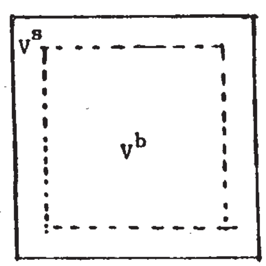

Fig. 2

1.3 (cont.)

3. Suppose there is a surface tension σ such that

$$dU = Tds - pdV + \sigma dA + \mu dN$$

Where μ is the chemical potential per atom. Also, $dU = dU^b + dU^s$, $dN = dN^b + dN^s$ etc.

Show that $dU^s = TdS^s - pdV^s + \mu dN^s + \sigma dA$ for an adiabatic, reversible expansion or contraction that conserves N.

4. One can show that the surface portion of the Gibb's potential $G = U + pV - TS$ satisfies

$$dG^s = -S^s\, dT + V^s\, dp + \sigma dA + \mu dN^s.$$

Derive (in the Gibbs convention $V_s = 0$) from this the Maxwell relation

$$\left(\frac{\partial S^s}{\partial N^s}\right)_{T,A} = -\left(\frac{\partial \mu}{\partial T}\right)_\Gamma \quad \text{where } \Gamma \equiv N^s/A \text{ is the surface gas}$$

density

(Possible Hint: Take a partial of dG^s with respect to N^s.)