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Physics 603 **HOMEWORK ASSIGNMENT #4** Spring 2013

Due date Thursday, March 14 [deadline on March 26]. *Reminder:* midterm March 12.

1. (10) P&B problem 4.3. Note that the probability distribution in question is the binomial distribution.

2.(7) P&B problem 4.5. Recall that $q = \ln \mathcal{Z}$, where $\mathcal{Z}$ is the grand partition function.

3.(7) P&B problem 4.6

4.(6) P&B problem 4.19

5.(10) Kardar problem 4.9 (cf. P&B 4.10 and 4.11). Kardar's $\mathcal{Q}$ is our $\mathcal{Z}$; $\mathcal{G}$ is Φ.

Langmuir isotherms: an ideal gas of particles is in contact with the surface of a catalyst.

(a) Show that the chemical potential of the gas particles is related to their temperature and pressure via $\mu = k_B T\left[\ln\left(P/T^{5/2}\right) + A_0\right]$, where A_0 is a constant.

(b) If there are $\mathcal{N}$ distinct adsorption sites on the surface, and each adsorbed particle gains an energy ϵ upon adsorption, calculate the grand partition function for the two-dimensional gas with a chemical potential μ.

(c) In equilibrium, the gas and surface particles are at the same temperature and chemical potential. Show that the fraction of occupied surface sites is then given by $f(T, P) = P/\left(P + P_0(T)\right)$. Find $P_0(T)$.

(d) In the grand canonical ensemble, the particle number N is a random variable. Calculate its characteristic function $\langle \exp(-\mathrm{i}kN)\rangle$ in terms of $\mathcal{Q}(\beta\mu)$, and hence show that

$$\langle N^m \rangle_c = -(k_B T)^{m-1} \left.\frac{\partial^m \mathcal{G}}{\partial \mu^m}\right|_T ,$$

where $\mathcal{G}$ is the grand potential.

(e) Using the characteristic function, show that

$$\left\langle N^2 \right\rangle_c = k_B T \left.\frac{\partial \langle N \rangle}{\partial \mu}\right|_T .$$

(f) Show that fluctuations in the number of adsorbed particles satisfy

$$\frac{\left\langle N^2 \right\rangle_c}{\langle N \rangle_c^2} = \frac{1 - f}{\mathcal{N} f} .$$