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Physics 603

HOMEWORK ASSIGNMENT #2

Spring 2013

Due date for problems on Thursday, Feb. 14 [deadline on Feb. 19].

1.(5) Consider a classical one-dimensional system of two noninteracting particles of the same mass m . The motion of the particles is restricted to a region of the x axis between $x = 0$ and $x = L > 0$, but otherwise they are not subjected to any potential (they are “free”). Let x_1 and x_2 be the position coordinates of the particles, and p_1 and p_2 be the canonically conjugated momenta. The total energy of the system is between $E - \Delta/2$ and $E + \Delta/2$. Draw the projection of phase space in a plane defined by position coordinates. Indicate the region of this plane that is accessible to the system. Draw similar graphs in the plane defined by the momentum coordinates.

2.(5) Calculate the number of accessible microscopic states of a system of two localized and independent quantum oscillators, with fundamental frequencies ω_0 and $3\omega_0$, respectively, and total energy $E = 10\hbar\omega_0$. (Recall that the energy of a quantum oscillator is a half-integer multiple of \hbar times its fundamental frequency; cf. P&B eq. 2.4.14.) What is the associated entropy?

3. (10) P&B 2.7. This is a generalization of the argument on P&B, pp. 34-35.

4. (10) A thermalized gas particle is suddenly confined to move in one dimension. Its mixed state is described by an initial density function $\rho(q, p, t = 0) = \delta(q)f(p)$, where $f(p) = h^{-1} \lambda_T \exp(-p^2/2mk_B T)$. (Think of this as an ensemble of 1D systems with the given $f(p)$.)

(a) Starting from Liouville's equation, derive $\rho(q, p, t)$ and sketch it in the (q, p) plane. The answer is a line with slope m/t . Note that it is not the trajectory at multiple times, which would be a horizontal line.

(b) Derive the expressions for the averages $\langle q^2 \rangle$ and $\langle p^2 \rangle$ and at $t > 0$. Note that the result is ballistic-like spreading rather than diffusive.

(c) Suppose that hard walls are placed at $q = \pm Q$. Describe $\rho(q, p, t \gg \tau)$, where τ is an appropriately large relaxation time. Argue that for $(4n-1)Q < t|p|/m < (4n+1)Q$ the motion is in the forward direction so that $p = +|p|$ while for $(4n+1)Q < t|p|/m < (4n+3)Q$ the motion is in the backwards direction so that $p = -|p|$. As time goes on, the slope decreases from that in part a, and one finds a family of parallel straight lines all with a slope that decreases as time increases. Show that the separation between these lines decreases like $2mQ/t$.

(d) A “coarse-grained” density $\tilde{\rho}$ is obtained by ignoring variations of ρ below some small resolution in the (q, p) plane; for example, by averaging ρ over cells of the resolution area. Find $\tilde{\rho}(q, p)$ for the situation in part (c), and show that it is stationary.

[This problem is from Kardar. Try it first yourself; as promised, I have added many hints, but if you have difficulties, you can find the solution in that book. If you do look at Kardar, look at his next problem dealing with the evolution of entropy, which is not assigned, starting (with minor modification) as:

Recall that the probability density $\rho(q, p; t)$ is a probability in phase space. The associated entropy is

$$S(t) = - \int d\Gamma \rho(q, p; t) \ln \rho(q, p; t)$$

(a) Show that if $\rho(q, p; t)$ satisfies Liouville's equation for a Hamiltonian \mathcal{H} , then $dS/dt = 0$. You might also look at subsequent parts for an example of the use of Lagrange multipliers.]