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Physics 603 **HOMEWORK ASSIGNMENT #1** Spring 2013

Due date for problems on Tuesday, Feb. 5 [deadline on Feb. 7].

1. a) Find the Legendre transform $\Psi(X)$ of $\Gamma(x) = x^3$.

b) Carry out an explicit Legendre transformation for a more complicated function: Consider the thermodynamic potential Γ:

$$\Gamma(w,x) = A + Bw + Cx^2 + Dw^2 + Ew^2x^2$$

Calculate $W = (\partial\Gamma/\partial w)_x$ and $X = (\partial\Gamma/\partial x)_w$

Construct explicitly the thermodynamic potential $\Psi(W,x)$ and from it verify the relations

$w = -(\partial\Psi/\partial W)_x$ and $X = (\partial\Psi/\partial x)_w$

2. a) Derive the Maxwell relation associated with H(S,p).

b) Verify the Maxwell relation $(\partial S/\partial V)_T = (\partial p/\partial T)_V$. From which thermodynamic function does it originate?

c) Extend U to $U(S,V,m)$ and G to $G(T,p,m)$ by adding $H\,dm$ (i.e. the magnetic work ON an object is $H\,dm$, analogous to $-p\,dV$ for mechanical work), and write down the new Maxwell relations involving H and/or m that result. (Note that this formulation includes the magnetic energy of the object; cf. the Kittel posting. Here m is taken as extensive, because M curiously is conventionally reserved for magnetization density (i.e., magnetization per volume). In older books, the extensive magnetization is often written 𝔐 [actually in Germanic Faktur, but Old English is the closest on my computer].)

3. (essentially Kardar 1-7) For an elastic filament it is found that, at a finite range in temperature, a displacement x requires a force

$$J = ax - bT + cTx ,$$

where a, b, and c are constants. Furthermore, its heat capacity at constant displacement is proportional to temperature: $C_x\,[= T\,\partial S/\partial T|_x] = A(x)\,T$.

a) Use an appropriate Maxwell relation to calculate $\partial S/\partial x|_T$.

b) Show that A must be independent of x: i.e., $dA/dx = 0$.

c) Calculate $S(T,x)$, assuming $S(0,0) = S_0$.

d) Show that the heat capacity at constant tension, $C_J = T\,\partial S/\partial T|_J$, can be written as

$$C_J = T\left[A + \frac{(ab - cJ)^2}{(a + cT)^3}\right]$$