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Physics 603 **HOMEWORK ASSIGNMENT #3** Spring 2012

Due date for problems on Tuesday, Feb. 21 [deadline on Feb. 23].

1. Calculate the number of accessible microscopic states of a system of two localized and independent quantum oscillators, with fundamental frequencies ω_0 and $3\omega_0$, respectively, and total energy E == $10\hbar\omega_0$. (Recall that the energy of a quantum oscillator is a half-integer multiple of $\hbar$ times its fundamental frequency; cf. P&B eq. 2.4.14.) What is the associated entropy?

2. Consider a classical one-dimensional system of two noninteracting particles of the same mass m. The motion of the particles is restricted to a region of the x axis between x = 0 and x = L > 0, but otherwise they are not subjected to any potential (they are "free"). Let x_l and x_2 be the position coordinates of the particles, and p_1 and p_2 be the canonically conjugated momenta. The total energy of the system is between $E–\Delta/2$ and $E + \Delta/2$. Draw the projection of phase space in a plane defined by position coordinates. Indicate the region of this plane that is accessible to the system. Draw similar graphs in the plane defined by the momentum coordinates.

3.A thermalized gas particle is suddenly confined to move in one dimension. Its mixed state is described by an initial density function $\rho(q, p, t = 0) = \delta(q)f(p)$, where $f(p) = h^{-1}\lambda_T \exp(-p^2/2mk_BT)$.

(a) Starting from Liouville's equation, derive $\rho(q, p, t)$ and sketch it in the (q, p) plane.

(b) Derive the expressions for the averages $\langle q^2\rangle$ and $\langle p^2\rangle$and at t > 0.

(c) Suppose that hard walls are placed at $q = \pm Q$. Describe $\rho(q, p, t \gg \tau)$, where τ is an appropriately large relaxation time.

(d) A "coarse-grained" density $\tilde{\rho}$ is obtained by ignoring variations of ρ below some small resolution in the (q,p) plane; for example, by averaging ρ over cells of the resolution area. Find $\tilde{\rho}$ (q,p) for the situation in part (c), and show that it is stationary.

[This problem is from Kardar. Try it first yourself, but if you have difficulties, you can find the solution in that book. If you do so, look at the following problem dealing with the evolution of entropy, which is not assigned, starting (with minor modification) as:

Recall that the probability density $\rho(\underset{\sim}{q}, \underset{\sim}{p}; t)$ is a probability in phase space. The associated entropy is

$$S(t) = -\int d\Gamma \rho(\underset{\sim}{q}, \underset{\sim}{p}; t) \ln \rho(\underset{\sim}{q}, \underset{\sim}{p}; t)$$

(a) Show that if $\rho(\underset{\sim}{q}, \underset{\sim}{p}; t)$ satisfies Liouville's equation for a Hamiltonian $\mathcal{H}$, then dS/dt = 0. You might also look at subsequent parts for an example of the use of Lagrange multipliers.]