Department of Physics
University of Maryland
College Park, MD 20742-4111

Physics 603 **HOMEWORK ASSIGNMENT #1** Spring 2012

Due date for problems on Tuesday, Feb. 7 [deadline on Feb. 9].

1. a) Show that the Legendre transform of $\Gamma(x) = \frac{1}{4} x^2$ is $\Delta(X) = -X^2$. Then follow the suggestion in the Callen excerpt and draw (using the program of your choice or by hand on graph paper if necessary) $\Gamma(x)$ and a large number (say 10–20) of straight lines with various slopes X and associated ordinate intercept $\Delta(X)$.

b) Carry out an explicit Legendre transformation for a more complicated function: Consider the thermodynamic potential Φ (what we had called Γ):

$$\Phi(w,x) = A + Bw + Cx^2 + Dw^2 + Ew^2x^2$$

Calculate $W = (\partial\Phi/\partial w)_x$ and $X = (\partial\Phi/\partial x)_w$

Construct explicitly the thermodynamic potential $\Psi(W,x)$ (analogous to Δ) and from it verify the relations

$w = -(\partial\Psi/\partial W)_x$ and $X = (\partial\Psi/\partial x)_w$

2. Verify that the assertion in class that $C_p - C_V > 0$.

a) Show this explicitly for a mole of ideal gas, for which $pV = RT$, where $R = k_B N_A$. (You should also use classical equipartition: $U = (f/2)\, n\, R\, T$, where f is the number of degrees of freedom [the number of quadratic expressions in the Hamiltonian] and n is N/N_A.)

b) Starting with $C_Y = T(\partial S/\partial T)_Y$, Y = p or V, show that $C_p - C_V = T\,(\partial S/\partial V)_T(\partial V/\partial T)_p$.
Hint: Start with S(T,V), find the differential dS, then plug it into the expression for C_Y.

Apply a Maxwell relation to $(\partial S/\partial V)_T$ and then show

$$C_p - C_V = -T\,\frac{\left(\dfrac{\partial V}{\partial T}\right)_p^2}{\left(\dfrac{\partial V}{\partial p}\right)_T} = -T\,\frac{\left(\dfrac{\partial p}{\partial T}\right)_V^2}{\left(\dfrac{\partial p}{\partial V}\right)_T}$$

3. a) Verify the Maxwell relation $(\partial T/\partial V)_S = -(\partial p/\partial S)_V$

b) Extend U to U (S,V,M) and G to G(T,p,M) by adding – B dM (i.e. the magnetic work ON an object is – B dM, analogous to – p dV for mechanical work), and write down the new Maxwell relations involving B and/or M that result.