Physics 603: Final Exam **Name (*print*): ___________________________________**

"I pledge on my honor that I have not given or received any unauthorized assistance on this examination."

May 13, 2013 ***Sign* Honor Pledge:____**

Be sure to look at all problems and do the easiest parts first. Budget your time and don't get bogged down on parts of problems that you find hard.

1. (7) a) For a given pressure and temperature, which of the 4 thermodynamic functions (energy U, enthalpy H, Helmholtz (F or A) or Gibbs (G) free energy) is minimized in equilibrium?

b) Which of these equals μN? Why not the others?

2. (19) Circle all of the following that have a low-temperature heat capacity $\propto \exp(-E_g/k_BT)$

a) Simple harmonic oscillator b) Einstein model of phonons in solid c) Debye model

d) Rotational motion of a dimer e) Translational motion of a dimer

f) Electronic states in a simple (free-electron) metal g) Superfluid ^{4}He & superconductors

For items a-e, give the (high temperature) equipartition value of the heat capacity C_V

3. (8) For N entities in d=3, give the (possibly unbounded or ∞) value of

a) Number of distinct **k** values in a solid

b) Number of distinct **k** values in an electron gas (T > 0)

c) Number of phonons

d) Number of spin-up electrons in a system with no magnetization

4. (15) We learned that the N-site lattice gas model $\mathcal{H} = -\varepsilon_{aa} \Sigma_{\langle i,j\rangle} n_i n_j = -\varepsilon_{aa} N_{aa}$ in a grand canonical ensemble for various numbers of atoms $N_a = \Sigma_i n_i$ is equivalent to the N-site Ising model $\mathcal{H} = -J \Sigma_{\langle i,j\rangle} \sigma_i \sigma_j - B \Sigma_i \sigma_i$ in a canonical ensemble. Each site has *q* nearest neighbors.

a) Check that $\sigma_i = 2 n_i - 1$and then use it to derive the nearest neighbor bond energy ε_{aa} and the chemical potential μ of the lattice gas model in terms of J and B.

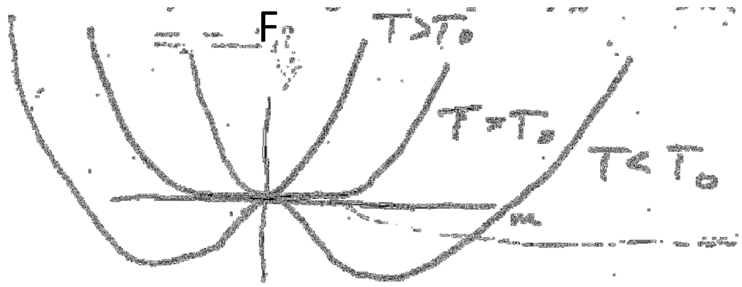
b) In mean field, write N_{aa} in terms of N and N_a. Is the actual value of N_{aa} larger, smaller, or the same as the mean-field value?

c) In the binary alloy qualifier/homework problem, x was defined as $(N_A - N_B)/N$. To what quantity in this problem does x correspond?

5. (19) In the Landau expansion, there are some special cases (in particular near what is called a "tricritical point") where $f_4 = 0$, while $f_6 > 0$, (and $f_0 = 0$ for simplicity) so that the expansion is

$$F(m; T, B) = -m B + (1/2)\, a\, (T - T_0)\, m^2 + (1/6)\, f_6\, m^6$$

a) This has a continuous transition (again for $B = 0$), with $T_c = T_0$; show that $m^2(T) = (a/f_6)^{1/2} (T_0 - T)^{1/2}$ for $T < T_0$.



b) Find $F(m(T); T, 0)$ and then $U(T)$.

c) Find the specific heat exponents α and α' above and below T_c, respectively.
[i.e. $C \propto (T - T_0)^{-\alpha}$ or $(T_0 - T)^{-\alpha'}$]

6. (19) For a free-electron gas in d=1, we know that the DOS $\mathcal{G}(\varepsilon) \propto \varepsilon^{-1/2}$.

a) Find the prefactor in terms of a power of ε_F, the number of electrons N and a number of order 1.

b) Find the ground-state (T=0) energy U(0) of a free-electron gas in dimension d=1.

c) Find the leading-order (in T) contribution to U(T) at $T \ll T_F$, again for d=1.

d) If a metal in d=3 is *compressed* under high pressure so that its volume decreases by 3%, what happens to its Fermi energy ε_F (and temperature T_F)? [No need to derive $\varepsilon_F(V)$.]

7. (23) Recall from class that for an ideal Bose gas in 3 dimensions (d=3)

$(N - N_0)/V = \lambda_T^{-3}\, \mathrm{Li}_{3/2}(z)$ where $\mathrm{Li}_\nu(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^\nu} = \left[\Gamma(\nu)\right]^{-1} \int_0^\infty \frac{x^{\nu-1}dx}{z^{-1}\exp(x) - 1}$

and so $\mathrm{Li}_\nu(1) = \zeta(\nu)$ [PB call $\mathrm{Li}_\nu(z)$ the Bose-Einstein function $g_\nu(z)$.]

a) Rewrite this expression for $(N-N_0)$ for free, non-interacting, non-relativistic bosons in arbitrary dimension d.

b) i) Rewrite this expression for $(N-N_0)$ for free, non-interacting, relativistic bosons ($\varepsilon \propto |k|$) in arbitrary dimension.

ii) Why is this expression not relevant for electrons in graphene for d=2?

c) For trapped atoms in d=3 we saw wrote that the DOS $\mathcal{G}(\varepsilon) \propto \varepsilon^2$.
i) Which of the attributes free, non-interacting, and/or non-relativistic is/are not applicable to this case? ii) What form does the expression for $N-N_0$ take?

d) If some system has the DOS $\mathcal{G}(\varepsilon) \propto \varepsilon^\rho$, what inequality must ρ satisfy for the system to undergo Bose-Einstein condensation? (Assume the system is pure and has no background potential, as in the treatment in class and the text.)

e) For a superfluid, which of the attributes free, non-interacting, and/or non-relativistic is/are not applicable? Give an example of a behavior of a superfluid that differs from an ideal bose gas.