Physics 603: Final Exam **Name (*print*):** ___________________________________

"I pledge on my honor that I have not given or received any unauthorized assistance on this examination."

May 14, 2012 ***Sign* Honor Pledge:**____

Be sure to look at all problems and do the easiest parts first. Budget your time and don't get bogged down on parts of problems that you find hard.

1. In the Landau expansion, there are some special cases (in particular near what is called a "tricritical point") where $f_4 = 0$, while $f_6 > 0$, so that the expansion is

$$\psi\,(m;\,T,\,B) = -\,m\,B + (1/2)\,a\,(T - T_0)\,m^2 + (1/6)\,f_6\,m^6$$

a) On a single graph sketch ψ (m; T, 0) for T above, at, and below T_0; label each curve clearly.

b) Show that this is a continuous transition (again for B = 0), with $T_c = T_0$.

c) i) Find $m^2(T)$ and then the magnetization exponent β. (Again, take B=0.)

ii) Is this value of β larger than, equal to, or smaller than β for the standard critical point we studied? Comment on the origin.

d) Find the magnetic susceptibility $\chi = \lim_{B\to 0}\,(m/B)$ above Tc. What is the associated exponent γ?

2. [Adapted from a qualifier problem] Consider a gas of N *indistinguishable* spinless non-relativistic particles of mass m in a volume V at temperature T.

a) What is the thermal wavelength λ_T? For what temperature range can the translational motion of the particles be treated using classical statistical mechanics (as opposed to quantum statistics)?

b) i) Suppose the particles interact with a "hard-core" potential $u(r) = \infty$ for $r < a$, and $u(r) = 0$ for $r > a$, as in a homework problem. Argue that the canonical partition function for each particle can be written $Z_1^a = [V - (N-1)(4\pi/3)a^3]/\,\lambda_T^3$. Assuming (not fully correctly, it turns out) that the N-particle canonical partition function Z_{tr} (say $Z_{N,tr}$) for the translational motion of the N particles could be obtained by combining the Z_1^a in the same way that we did in the independent-particle ideal-gas case, write down $Z_{N,tr}$.

ii) From $Z_{N,tr}$ find F, U, and p. (Assume that $N \gg 1$ so that $N-1 \approx N$.) Compare the equation of state, obtained from your result for p, with the van der Waals equation of state and, for $a \to 0$, the ideal gas equation.

iii) Expand the equation of state to find the leading virial coefficient $B_2(T)$ and compare it to the correct result $(2\pi/3)a^3$ obtained in the homework problem. Thus, the approximations in part b-i lead to a good qualitative picture but are not quantitatively accurate.

(If you have time at the end of the exam, you can look for a clue earlier in part b that something is amiss.)

3. Suppose that one could write the Gibbs free energy $G(T_0,p) = A(T_0)p^4$, where A depends on T but not p; take it to be some constant. You do not need to write T_0 as you proceed.

a) What is $V(T_0,p)$, i.e. $V(p)$?

b) Draw the graph of G vs. p to find the Helmholtz free energy F(V) (i.e. $F(T_0,V)$) using a Legendre transformation. Then do the algebra to get F(V).

4. Recall from class that for an ideal Bose gas in 3 dimensions (3D)

$(N - N_0)/V = \lambda_T^{-3}\, \mathrm{Li}_{3/2}(z)$ where $\mathrm{Li}_\nu(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^\nu} = \left[\Gamma(\nu)\right]^{-1} \int_0^\infty \frac{x^{\nu-1}dx}{z^{-1}\exp(x)-1}$

and so $\mathrm{Li}_\nu(1) = \zeta(\nu)$ [PB call $\mathrm{Li}_\nu(z)$ the Bose-Einstein function $g_\nu(z)$.] Also, for particles with dispersion relation $\varepsilon \propto |k|^s$ in n dimensions, the density of states $g(\varepsilon) \propto \varepsilon^{(n/s)-1}$.

a) i) What property of $\mathrm{Li}_\nu(z)$ allows Bose-Einstein condensation (BEC) to occur in 3D but not in 2D for non-relativistic particles.

ii) Is there BEC in 1D? Justify your answer very briefly (so that it is clear you are not just guessing!).

b) Now consider relativistic particles, with $\varepsilon \propto |k|$. Is there BEC in 3D? in 2D? in 1D? Justify your answer very briefly.

c) Why is there no BEC for photons in a blackbody oven or phonons in a crystal?

5. a) For an ideal gas in 3D (three dimensions) at T=0, find the total value of ε^2 (call it Υ) of all the N fermions [electrons]. (Note that this is similar to finding U(T=0) as the total value of ε.) What is $\Upsilon/(N\varepsilon_F^2)$?

b) In 3D find the leading-order correction to the value of Υ at finite T. Use the Sommerfeld expansion or one of the corollary forms of it we derived.

c) Which of the following depend on dimension? i) $\Upsilon/(N\varepsilon_F^2)$ at T=0

ii) Power of k_BT in the leading order correction at finite T

iii) Value of the prefactor to the k_BT part of the leading order correction at finite T