Physics 601
Dr. Dragt
Fall 2002

Reading Assignment #4:

1. Dragt

    (a) Sections 1.5 and 1.6 of Chapter 1 (Introductory Concepts).

    (b) Notes VI, Hamilton's Equations of Motion (to be found right after
        the ELEMENTARY CONCEPTS Section).

2. Goldstein

    (a) Sections 3.7 through 3.12.

    (b) Sections 8.1 through 8.3. and Sections 8.5 through 8.6

Problem Set 4 due Monday, 10/14/02

34. Find the orbit $\boldsymbol{r}(t)$ for a nonrelativistic particle of mass m and charge
    q moving in spatially uniform and temporally constant electric and
    magnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$.

35. You are scanning, with stereo views, tracks in bubble chamber pho-
    tographs. A spiral track sketched below catches your eye, and you
    measure its coordinates at points a, b, and c:

Point	Coordinates in cm		
	x	y	z
a	3.84	.38	1.9
b	3.57	1.39	3.1
c	3.14	2.24	?

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The chamber is a uniform magnetic field

$$\boldsymbol{B} = B\boldsymbol{e}_z$$

With $B = 9000$ gauss. Assume that the particle charge q is equal in magnitude to the proton charge. Find the sign of the charge, and the momentum $\boldsymbol{p}_a$ the particle had when it was at point a. (Assume that the particle loses negligible energy due to scattering as it moves between points a and c. But, just from looking at the picture, how do you know it moved from a to c, and not from c to a?) Your answer should be a vector!

36. A velocity selector for a beam of charged particles of mass m, charge e, is to be designed to select particles of a particular velocity v_0. The velocity selector utilizes a uniform electric field E in the x-direction and a uniform magnetic field B in the y-direction. The beam emerges from a narrow slit along the y-axis and travels in the z-direction. After passing through the crossed fields for a distance ℓ, the beam passes through a second slit parallel to the first and also in the yz-plane. The fields E and B are chosen so that particles with the proper velocity moving parallel to the z-axis experience no net force.

   (a) If a particle leaves the origin with a velocity v_0 at a small angle with the z-axis, find the point at which it arrives at the plane $z = \ell$. Assume that the initial angle is small enough so that second-order terms in the angle may be neglected.

   (b) What is the best choice of E, B in order that as large a fraction as possible of the particles with velocity v_0 arrive at the second slit, while particles of other velocities miss the slit as far as possible?

   (c) If the slit width is h, what is the maximum velocity deviation δv from v_0 for which a particle moving initially along the z-axis can pass through the second slit? Assume that E, B have the values chosen in part (b).

37. A particle with charge q and mass m moves in the field of a magnetic dipole with moment $\boldsymbol{\mu}$. (This is a slightly simplified model of the Van Allen radiation.) The assumed magnetic field can be derived from a

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vector potential
$$\boldsymbol{A} = \frac{\boldsymbol{\mu} \times \boldsymbol{r}}{r^3}$$
where $\boldsymbol{r}$ is measured from the center of the dipole. Let the z axis be taken along $\boldsymbol{\mu}$.

(a) Find the Lagrangian of the particle in cylindrical coordinates ρ, ϕ, z. For simplicity, assume the motion is nonrelativistic.

(b) Find the equations of motion.

(c) Find the Hamiltonian and show that it is a constant of motion.

(d) Find a constant of motion related to the axial symmetry of the problem. Show that if $\rho(t)$ and $z(t)$ are known, $\phi(t)$ can be found by an integration.

(e) Exhibit a Lagrangian which describes the motion in the ρ-z coordinates.

38. A particle of mass m moves under the influence of gravity on the inner surface of a paraboloid of revolution $x^2 + y^2 = az$ which is assumed frictionless.

(a) Introduce cylindrical coordinates ρ, ϕ, z. Write a Lagrangian for the system employing ρ and ϕ as generalized coordinates.

(b) Find the Hamiltonian for the system.

(c) Find two constants of motion.

(d) Show that the particle will describe a horizontal circle in the plane $z = h$ provided that it is given the proper angular velocity ω. What is the magnitude of this velocity?

(e) Obtain the "frequency of oscillation" in ρ for orbits near the orbit of part d.

39. A bead of mass m is free to slide without friction on a rotating hoop of radius a in the presence of gravity. See the accompanying figure.

(a) Find the Lagrangian using θ as a generalized coordinate.

(b) Find the Hamiltonian H.

(c) Show that H is a constant of motion if $\dot{\omega} = 0$, but the energy is not.

(d) Study the stability of the equilibrium solution $\theta = 0$ assuming $\dot{\omega} = 0$. Find the frequency Ω of small oscillations about $\theta = 0$, again assuming $\dot{\omega} = 0$. Show that if ω is increased to the point where $\theta = 0$ becomes an unstable equilibrium point, then two other equilibrium points appear.

40. Show that the two Lagrangians

$$L_1 = \frac{1}{2}(\dot{q}^2 - q^2)$$

and

$$L_2 = \left[\frac{1}{2}q\dot{q}^2 - \frac{1}{3}q^3\right]\sin t + \frac{1}{6}\dot{q}^3 \cos t$$

give the *same* equation of motion. Do L_1 and L_2 differ by a total time derivative, i.e. a term of the form $\frac{d}{dt}f(q,t)$? This problem illustrates the nonuniqueness of Lagrangians.

41. A system with 3 degrees of freedom has the peculiar Lagrangian

$$\begin{aligned} L(q,\dot{q},t) &= \dot{q}_1^2 q_2 \cos q_3 + \dot{q}_2^2 q_2 q_3 + \dot{q}_3^2 \tan(q_2 q_3) \\ &+ \dot{q}_1 \dot{q}_2 \dot{q}_3 + q_2 \exp q_3 + ct^5 q_2 q_3. \end{aligned}$$

(a) Find a constant of motion.

(b) If $c = 0$, find an additional constant of motion.

42. The system shown in the figure below is used in the construction of gravimeters.

A mass M hangs from a beam B of length a and of negligible mass. This beam can rotate freely at C in the $x = 0$ plane. To counteract the gravity force acting on M , a spring S is used. The restoring force provided is $k(\ell - \ell_0)$ where ℓ_0 is the natural length of the spring, and k is the spring constant.

(a) Using a Lagrangian, find the equation of motion for the angle θ, and find the equilibrium angle θ_0.

(b) Assume that the mass M is such that the equilibrium angle is given by $\theta_0 = 0$. Consider small oscillations about equilibrium. Show that the frequency ω of oscillation is proportional to a fractional power of ℓ_0, and thus can be made very small by proper construction of the spring. (This property leads to the high sensitivity of gravimeters to small change in the acceleration of gravity.)

(c) This system could also be used to isolate the mass M from vertical noise in the support. Assume that the supporting wall which contains points C and A moves up and down (vertically) with amplitude Λ and angular frequency Ω. Incorporate this fact into the Lagrangian to find the appropriate equation of motion. Describe the transfer of noise to the mass M as a function of the frequency of the noise and the resonance frequency of the mass-spring system. That is, find the ratio of the amplitude of M to that at C as a function of frequency.

43. The Implicit Function Theorem states that if the Jacobian determinant of a mapping is not zero in some region R, then the mapping is locally invertible. That is, for each point in R there is some region R', containing the point, within which the map is one to one. The region R' may be smaller than R. Suppose the Jacobian determinant is different from zero everywhere. Then one might wonder whether or not the map has to be globally one to one. The answer is, "not necessarily so", as the following example shows. Let f be the function

$$f = e^x \sin y.$$

Define the mapping $\{x, y\} \to \{u, v\}$ by the rule

$$u = \partial f/\partial y \ , \ \ v = \partial f/\partial x.$$

(a) Verify that the Jacobian determinant for this mapping is nonzero everywhere in the finite x, y plane.

(b) Show that the mapping is not globally one to one by examining the image of the x, y plane in the u, v plane.