Physics 601
Dr. Dragt
Fall 2002

Reading Assignment #12:

1. Dragt
   (a) Get caught up on past reading assignments.
2. Goldstein
   (a) Chapter 10.

Problem Set 12 due Friday, 12/13/02

101. Suppose a "burst" of protons is injected into a uniform electric field $\boldsymbol{E} = E_0 \boldsymbol{e}_z$. Assume the burst is initially concentrated at x and $y = 0$ and v_x and $v_y = 0$, but is uniformally spread in z and v_z about the values $z = 0$ and $v_z = v_z^0$ within intervals $\pm\Delta z$ and $\pm\Delta v_z$. Thus the problem is essentially that of one dimensional motion along the z axis. The initial distribution is shown schematically below.

Find the distribution at later times, and verify Liouville's theorem. Do not assume Δz and Δv_z are infinitesimal. Neglect Coulomb interactions between particles.

102. Consider a free particle of mass m in 3-dimensional space. Study the Poisson bracket Lie algebra generated by the dynamical variables L_x, L_y, L_x and p_x, p_y, p_z where $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$. Compare this algebra to that of the Euclidean Group.

103. Consider a Hamiltonian of the form $\mathcal{H}(p, q, t) = \sum_{m,n=0}^{\infty} a_{mn}(t)(p-b)^m(q-c)^n$ where b and c are constants. This is a pretty general form. Show that the transformation $P = 2p$ and $Q = 2q$ is *not* canonical, but that nevertheless there exists a new Hamiltonian $K(P, Q, t)$ such that $\dot{Q} = \frac{\partial K}{\partial P}$, $\dot{P} = -\frac{\partial K}{\partial Q}$. Find K.

104. Suppose M is a 2×2 matrix. Show that the necessary and sufficient condition for M to be symplectic is $\det(M) = 1$.

105. Let M denote the $2n \times 2n$ matrix

$$M = \begin{pmatrix} R & 0 \\ 0 & R \end{pmatrix}$$

where 0 denotes an $n \times n$ zero matrix and R denotes an $n \times n$ orthogonal matrix. Show that M is symplectic.

106. Let α and β be *any* two $2n$ component real vectors different from zero. Show that there exists a symplectic matrix M such that $\beta = M\alpha$. You have shown that the symplectic group acts transitively on the manifold ($E^{2n}-$ origin).

107. Goldstein 9.4.

108. Goldstein 9.39.

All problems are worth 10 points each.