Physics 601
Dr. Dragt
Fall 2002

Reading Assignment #11:

1. Dragt

   (a) Section 1.7 on Defintion of Poisson Bracket.

   (b) Sections 3.1, 3.2, 3.3, 3.4, 3.6.1, and 3.7 of Chapter 3 on Symplectic
       Matrices.

2. Goldstein

   (a) Chapter 9.

Problem Set 11 due Friday, 12/6/02

92. Goldstein 6.4.

93. Three discs, each of mass M and radius R, are supported with equal
    separation along a slender rod to which they are rigidly fastened at
    their centers as shown.

    (a) Find the normal modes and normal frequencies. Make sure you
        get the potential energy right. The torsion constant for the rod
        is "k", i.e., for a twist of θ radians per unit length a torque of
        magnitude $k\theta$ is required.

(b) Suppose motion is initiated by a torsional impulse instantaneously applied to an end disc, which imparts an angular momentum J to it. Give an expression showing the angular displacement of the center disc as a function of time.

94. (Extra Credit) You are given a uniform thin equilateral triangular plate having mass m and side a.

   (a) Find the moment of inertia tensor for the coordinate system shown below. Do you understand physically why $I_{11} = I_{22}$?
      The origin is at the C.M. e_3^* is out of the page.

   (b) Suppose the triangle is suspended by three identical springs (having spring constant k with negligible natural length) as shown below. The plate is free (with the springs being stretched or compressed) to move in 3 dimensional space. Thus the system has 6 degrees of freedom. Find the normal frequencies and normal modes for small oscillations about the equilibrium position. Neglect gravity.

95. Three identical capacitors and three identical inductors ae connected as shown. Find the resonant frequencies of the system.

96. Show that the most general form of the solution to the one-dimensional wave equation is $f(x - vt) + g(x + vt)$ where f and g are arbitrary functions. At $t = 0$ a string of length L, tension τ, and density ρ has a pulse propagating on it to the right. The pulse has the shape shown below.

All angles are $45°$, $90°$, or $135°$. Assign some dimensions to the pulse. Using your dimensions

   (a) Sketch the pulse configuration at the time t_A when the disturbance at point A has reached the right wall.

   (b) Similarly, sketch the pulse configurations at times t_B and t_C.

   (c) Supply the mathematical reasoning required to justify your sketches.

3

97. Consider the array of N identical masses m and $N+1$ identical springs (with constant k) shown below.

Considering only the compressional motions illustrated, find the normal frequencies and modes.

98. At $t = 0$ a string of ∞ length, tension τ, and density ρ satisfies the initial conditions

$$q(x,0) = Ae^{-\frac{x^2}{B}}, \qquad\qquad (A \text{ and } B \text{ are constants})$$

$$(\partial q/\partial t)|_{t=0} = 0.$$

Find $q(x,t)$ for all t.

99. Consider a "plucked" string initially released with zero velocity from the configuration shown below. The string has density (mass per unit length) ρ and is under a tension τ. Find the string's subsequent motion as a Fourier series, and compute the relative strengths of the various harmonic frequencies. Sum the series, and draw pictures of the string configuration at later times. Compute the total energy of the excited string and compare this energy to that required to deform the string from its equilibrium condition to the "plucked" initial condition.

4

100. Repeat problem 99 for a *struck* string. That is, assume the initial conditions

$$q(x, 0) = 0 \ , \ (\partial q / \partial t)|_{t=0} = vD\delta(x - L/2).$$

$$(v^2 = \tau / \rho).$$

Problem 94, which is optional, is worth 30 points. All the rest are worth 10 points each.