Physics 601
Dr. Dragt
Fall 2002

Reading Assignment #10:

1. Dragt
   (a) Notes on Small Oscillations.
2. Goldstein
   (a) Chapter 6.
   (b) Chapter 13.

Problem Set 10 due Wednesday, 11/27/02

83. Find the moment of inertia tensor for the parallelepiped shown below. It has mass M, sides a, b, and c, and the origin is at one corner as shown below.

84. Which statements below are true and which are false, and why?

(a) Select any point P in or outside a rigid body. Then, no matter how irregularly shaped and weighted the body, there will always exist three orthonormal principal axis through P which are eigenvectors of the moment of inertia tensor.

(b) Any axis through the center of mass of a cube of uniform density is a principal axis.

(c) Any axis through the center of mass of a regular tetrahedron of uniform density is a principal axis.

(d) The moment of inertia about any axis through the center of a homogeneous sphere of mass M and radius R is MR^2.

(e) Any axis through any point of a regular tetrahedron of uniform density is a principle axis.

(f) Any axis through any point of a homogeneous sphere is a principal axis.

85. Consider an "oscillator" consisting of a disk of mass M and radius R which rolls *without slipping* on a horizontal plane and experiences restoring forces due to 2 springs. See the sketch below:

Each spring has negligible natural length, and spring constant k. They are attached to the disk at its center by a low friction bearing.

(a) Write the Lagrangian for the system assuming that the horizontal coordinate q is measured from the equilibrium position.

(b) Find the frequency of small oscillations about equilibrium.

86. Rube Goldberg once made a plane pendulum with a sphere at one end and a cube at the other. See the picture below. What was its period for small oscillations? The sphere and cube were made of material with density ρ.

87. Suppose that a rigid body has a moment of inertia tensor $I_{i,j}$. Show that no matter how the body fixed axes are chosen, it must always be true that

(a) $I_{i,i} \geq 0$, $i = 1, 2$, or 3.

(b) $I_{1,1} + I_{2,2} \geq I_{3,3}$ (and cyclic permutations thereof).

(c) It is claimed that an oddly shaped block of quintessence has a moment of inertia tensor given (in elvish units) by

$$I = \begin{pmatrix} 2 & 4 & 0 \\ 4 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

What is your reaction?

88. Suppose a disc of mass M, radius R, and thickness d, rotates with angular velocity $\dot{\alpha}$ about its axis, while its axis in turn rotates with angular velocity $\dot{\beta}$ as shown below. Let $\boldsymbol{e}_1^*$ be along the axis of the disc, and measure α,β in such a way that the body fixed and space fixed axes coincide when $\alpha = \beta = 0$. Suppose α and β have the time dependencies $\alpha(t) = at$, $\beta(t) = bt + ct^2$.

(a) Compute the *body* fixed components of $\boldsymbol{\omega}$.

(b) Compute the moment of inertia tensor of the disc about the fixed point of the disc.

(c) Compute the kinetic energy $T(t)$ of the disc.

(d) Compute the *body* fixed components $N_j^*(t)$ of the torque required to maintain the specified motion.

89. A uniform sphere of mass M and radius R is cut exactly in half. One half is discarded, and the other half is placed on a rough table as shown below. Find the frequency for small oscillations about the equilibrium position.

90. Using the Euler equations show, as illustrated with a spinning book in class, that motion about $\boldsymbol{e}_2^*$ is unstable. [The principal axes $\boldsymbol{e}_i^*$ are ordered such that $I_1 < I_2 < I_3$.] Show that motion about either of the other two axes is stable. That is, show that if at $t = 0$ the initial conditions are such that $\omega_2^* \neq 0$ and ω_3^*, ω_1^* are of order ϵ, then this condition *does not* persist. On the other hand if $\omega_1^* \neq 0$, and ω_2^*, $\omega_3^* \approx \epsilon$, then this condition does persist, etc.

91. Review Problem DCM 1.16 already given as Problem 26 in Problem Set #3. Find Ω in terms of ω.

All problems are worth 10 points each.