Physics 601 Homework 8---Due Friday November 5

1. Consider a two-body system with reduced mass μ and a potential of the form $V = -ar^{-k}$ for $a,k > 0$ (*a,k* real).
    a. Show that circular orbits exist for any $k \neq 2$ and find the relationship between the radius r_0 and *L*.
    b. Linearize the equation of motion for *r* around r_0 and
        i. Show that stable orbits only exist for $k < 2$
        ii. Find the oscillation frequency for fluctuations in *r* for *k<2*.
        iii. Find the values of k for which the orbits close.

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3. For the by the Hamiltonian $H = \frac{\vec{p}^2}{2\mu} - \frac{k}{r^2}$ with k>0 (that is an attractive Coulomb or gravitational system) show that
    a. The Runge-Lenz vector defined by $\vec{A} = \vec{p} \times \vec{L} - \alpha\mu\hat{r}$ (where $\hat{r} = \vec{x}/r$) satisfies $[\vec{A}, H]_{PB} = 0$. Do this by explicit evaluation of the Poisson bracket. This means $\vec{A}$ is conserved
    b. The Runge-Lenz vector is in the plane of the orbit

4. Show by explicit evaluation that
    a. $[A_x, L_y]_{PB} = A_z$
    b. $[A_x, A_y] = -2\mu E\, L_z$

where $\vec{A}$ is the Runge-Lenz vector. If one includes cyclic permutation this yields the Lie algebra discussed in class.

5. Consider an elliptical orbit in the Kepler problem ($V(r) = -k/r$) with energy, *E*, and orbital angular momentum *ℓ*. Find $\overline{r^{-1}}$, $\overline{r^{-2}}$ and $\overline{\dot{r}^2}$ where the bar means time average. You may find the viral theorem helpful but you will need more input then that.

6. Suppose one has a "hard sphere" of radius *R* off of which a particle scatters via specular reflection---that is angle of incidence = angle of reflection on the plane tangent to the sphere. Find the differential cross-section.

7. Consider the potential $V(r) = \alpha/r^2$. Find the differential cross-section.