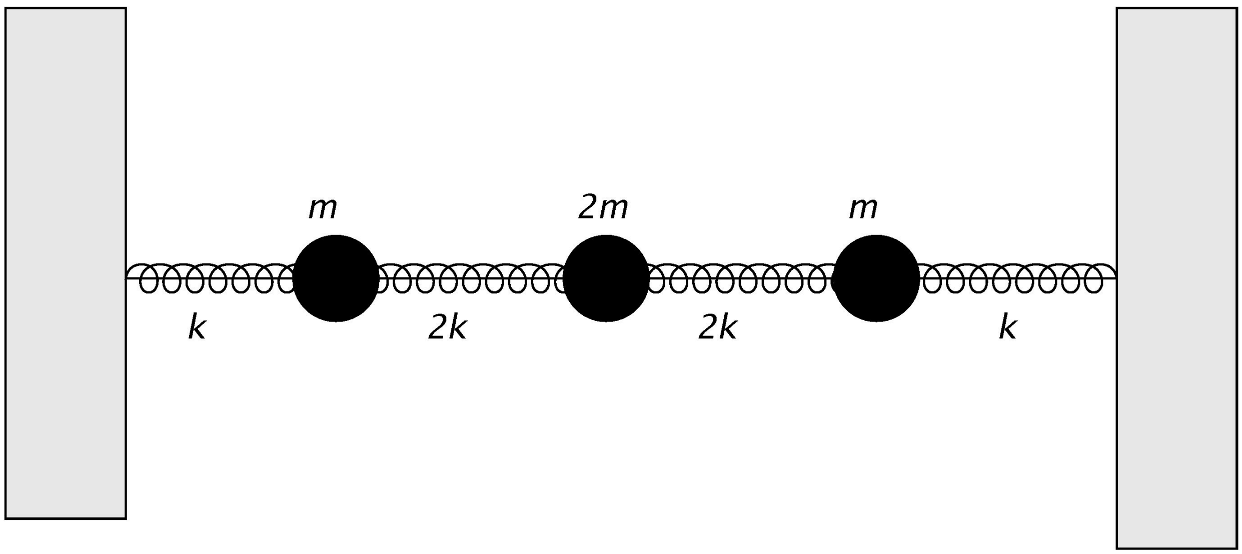
Physics 601 Homework 7---Due Friday October 29

You may find Mathematica very helpful for some of these problems!!

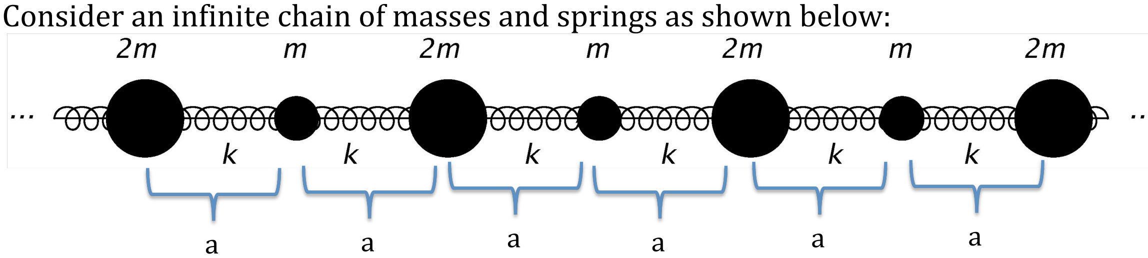
1. Three beads are free to move along a wire. They are connected two immovable walls by 4 springs. The masses and spring constants are as indicated on the figure.



   a. Find the normal modes and their associated frequencies by finding the eigenvectors and eigenvalues of the appropriate matrix.
   b. Verify that the modes are orthogonal with respect to the mass matrix.
   c. Suppose at t=0 all three particles are in their equilibrium positions with the two particle on the end at rest and the one in the middle moving with velocity v. (This can happens as a result of an impulse acting on the middle particle). Find the motion of the three particles.

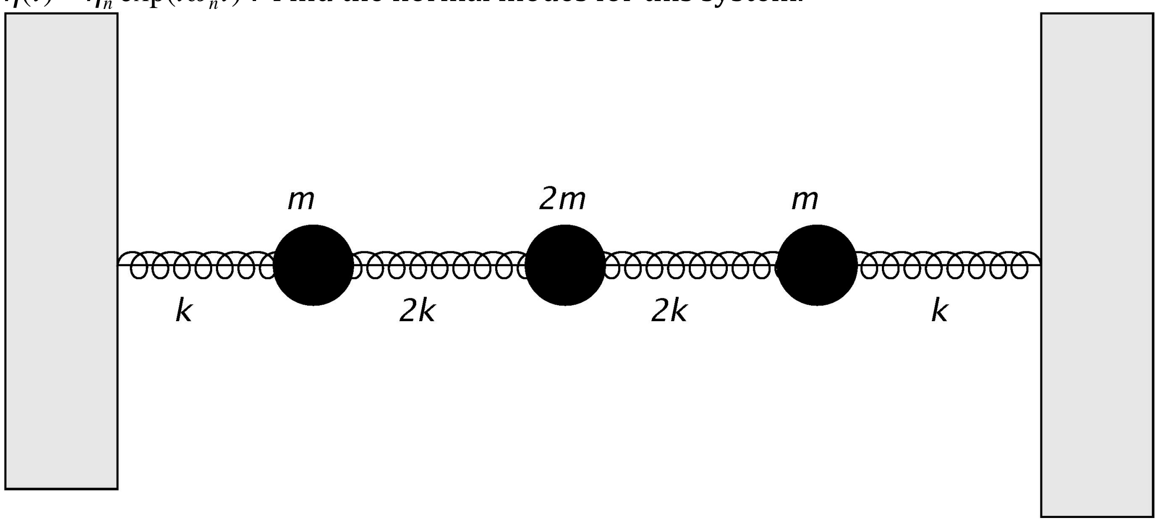
2. Consider the system described in the previous problem. Suppose that all the particles begin at rest in equilibrium a force given by $f(t) = f_0 \sinh(t/T)\theta(t)$ acts on the middle particle. Use Green's functions to determine the motion of the three particles.

3. Consider an infinite chain of masses and springs as shown below:



The masses alternate between two types, one twice as heavy as the other. The springs connecting the masses are all equal and in equilibrium the masses are all a distance *a* apart. The masses are constrained to move longitudinally. Find the dispersion relation relating frequency to wave number.

4. Three beads are free to move along a wire. They are connected two immovable walls by 4 springs. The masses and spring constants are as indicated on the figure. The third particle is charged and has a mass q. The first particle is attached to a shock absorber with a force given by $F = -m\omega_0 \dot{x}$ where $\omega_0^2 = \frac{k}{m}$. Using the phase-space formalism derived in class: $\vec{\eta} = \begin{pmatrix} \vec{q} \\ \vec{p} \end{pmatrix}$ $\vec{\vec{W}} = \begin{pmatrix} 0 & (\vec{\vec{T}})^{-1} \\ -\vec{\vec{V}} & -\vec{\vec{F}}(\vec{\vec{T}})^{-1} \end{pmatrix}$ with the equation of motion case given by $\dot{\vec{\eta}} = \vec{\vec{W}}\vec{\eta}$; normal modes are of the form $\vec{\eta}(t) = \vec{\eta}_n \exp(i\omega_n t)$. Find the normal modes for this system.



5. Suppose the system described in the problem above has the middle particle charged (with charge q) and is place in an electric field, oriented to the right with a magnitude of $E = E_0 \cos(2\omega_0 t)$. Find the positions of each mass as a function of time, assuming that the system has reached steady state.

6. Consider a driven damped system of oscillators problem using the phase-space formalism derived in class: $\vec{\eta} = \begin{pmatrix} \vec{q} \\ \vec{p} \end{pmatrix}$ $\vec{\vec{W}} = \begin{pmatrix} 0 & (\vec{\vec{T}})^{-1} \\ -\vec{\vec{V}} & -\vec{\vec{F}}(\vec{\vec{T}})^{-1} \end{pmatrix}$ with the equation of motion for the undriven case given by $\dot{\vec{\eta}} = \vec{\vec{W}}\vec{\eta}$; normal modes are of the form $\vec{\eta}(t) = \vec{\eta}_n \exp(i\omega_n t)$ and satisfy the eigenvalue equation $i\omega_n \vec{\eta}_n = \vec{\vec{W}}\vec{\eta}_n$. Now the driven oscillator of the form $\dot{\vec{\eta}} = \vec{\vec{W}}\vec{\eta} + \vec{f}(t)$ with $\vec{f}(t) = \begin{pmatrix} 0 \\ \vec{F}(t) \end{pmatrix}$. The Green's function for this system is a matrix valued equation of the form $\left(\frac{d}{dt}\vec{\vec{1}} - \vec{\vec{W}}\right)\vec{\vec{G}}(t-t') = \delta(t-t')\vec{\vec{1}}$.

a. Show that the equation of motion is solved by $\vec{\eta}(t) = \int_{-\infty}^{t} dt' \overleftrightarrow{G}(t-t')\vec{f}(t)$.

b. The Green's function depends on the boundary conditions. For the case where $\overleftrightarrow{G}(t-t') = 0$ for all t less than t' show that $\overleftrightarrow{G}(t-t') = 0$
$\overleftrightarrow{G}(t-t') = \exp\left(\overleftrightarrow{W}(t-t')\right)\theta(t-t')$

c. Writing $\overleftrightarrow{W} = \overleftrightarrow{S}^{-1}\overleftrightarrow{W}_d\overleftrightarrow{S}$ where $\overleftrightarrow{S}$ is the similarity matrix which diagonalizes $\overleftrightarrow{W}$ and $\overleftrightarrow{W}_d$ is the diagonal matrix of eigenvalues of $\overleftrightarrow{W}$ show that
$\overleftrightarrow{G}(t-t') = \overleftrightarrow{S}^{-1}\exp\left(\overleftrightarrow{W}_d(t-t')\right)\overleftrightarrow{S}\,\theta(t-t')$