Physics 601 Homework 11---Due Friday Dec. 3

1. In class we algebraically derived the expression for ω_z^{body} in terms of Euler angles and there time derivatives. Using the same methods derive the expression for ω_x^{body} and ω_y^{body}.
2. Consider a symmetric top where the two identical moments of inertia are given by I_0 and the other moment is by $I_0/2$. The (conserved) angular momentum about the body fixed z-axis is given by $I_0\omega_0$. Suppose that $mgl = \omega_0^2 I_0$.
   a. Suppose that the system precesses without nutating with a precession frequency of $2\,\omega_0$ Find the angle θ.
   b. Suppose that at t=0 the angle θ is slightly perturbed away from the equilibrium angle found in a. by an amount Δθ. Find the nutation frequency.
3. Consider the application of the formal short time expansion developed in class to a simple Harmonic oscillator: $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}\lambda^2 m\omega_0^2 x^2$ with $x = x_0 + \lambda x_1 + \lambda^2 x_2 + \ldots$. Suppose the initial conditions are fixed by $x_0(0) = x(0); \dot{x}_0(0) = 0; x_1(0) = 0, \dot{x}_1(0) = \dot{x}(0)$ with $x_n(0) = 0, \dot{x}_n(0) = 0$ for $n>1$. λ is taken to unity at the end of the problem.
   a. Solve for $x(t)$ to 5th order.
   b. Verify that the solution is identical to the exact solution Taylor expended in time up to t^5.
4. In class we formally developed the short-time expansion for a system with one degree of freedom. In this problem I would like you to use the same methods to derive it for 2 degrees of freedom : $L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - \lambda^2 V(x,y)$ with $x = x_0 + \lambda x_1 + \lambda^2 x_2 + \ldots ; y = y_0 + \lambda y_1 + \lambda^2 y_2 + \ldots$. Choosing sensible boundary conditions find an expression *x(t),y(t)* up to 4th order. The result should be given in terms of the initial position, the initial velocity, and partial derivatives with respect to *V* evaluated at the initial position.

5. Apart from short time expansion there is a context in which one can build an expansion in which the full potential is treated perturbatively: if one is working in a regime in which potential energy difference are always much less then the kinetic energy the approximation can be justified up to comparative long times. Consider the case of a physical pendulum whose Lagrangian is given by $L = \frac{1}{2}I\dot{\theta}^2 + V_0\cos(\theta)$ where *I* is the moment of inertia and V_0 is the maximum value of the potential energy (i.e. *m g L*) where L is length from the pivot point to the center of mass. The problem under consideration is this: suppose that at t=0 the system is at the minimum of the potential (θ =0) with an initial angular velocity $\dot{\theta} = \omega_0$.

   To develop the expansion insert powers of λ: $L = \frac{1}{2}I\dot{\theta}^2 + \lambda V_0\cos(\theta)$ with $\theta = \omega_0 t + \lambda\theta_1 + \lambda^2\theta_2 + \ldots$ and impose boundary conditions $\theta_i(0) = 0,\ \dot{\theta}_i(0) = 0$ for i. These boundary conditions are designed to ensure that the boundary conditions to our problem is solved

Find explicitly the form for θ_1 and θ_2.

6. This is a continuation of the problem discussed above. While the solution found there is formally correct based on the expansion, as with naïve perturbation theory for an anharmonic oscillator it has a problem with periodicity and a reorganized series can give much more accurate answers..
   a. Show on general grounds that for this problem that in the regime where system has sufficient energy to go "over the top" that θ(t) is of the form $\theta(t) = \omega t + \sum c_n \sin(n\omega t)$, where ω is in general not ω_0.

   b. Using the power counting scheme $\omega = \omega_0 + \lambda\omega_1 + \lambda^2\omega_2 + \ldots$, $c_n = \lambda^n (c_n^{(0)} + \lambda c_n^{(1)} + \lambda^2 c_n^{(2)} \ldots)$ compute $\theta(t)$ up to 2nd order.

7. This is a continuation of problems 4 and 5. Consider initial conditions where the total energy E is $10V_0$. Numerically solve the equations of motion. Plot the exaction solution and the 0th , 1st , 2nd order solutions for the expansions developed in 4 and 5. Briefly discuss what these plots tell you about the utility of these expansions.