Physics 601 Homework 3---Due Friday September 24

Goldstein problems 8.1, 8.6, 8.9, 8.26, 8.23

In addition:

1. For a particle in 3 dimensions the angular momentum operator is given by $\vec{L} = \vec{x} \times \vec{p}$ where $\vec{x}$ and $\vec{p}$ satisfy are canonical (*i.e.* they satisfy the canonical Poisson bracket relations.
    a. Show that $[L_i, L_j]_{PB} = \varepsilon_{ijk} L_k$ where *i,j,k* take on the values *x,y,z.* (Note that this is isomorphic to the commutators for the angular momentum in quantum mechanics. For those with mathematically inclinations, this is the Lie algebra SO(3).)
    b. Show that $[\vec{L}, L^2]_{PB} = 0$ (that is that $[L_i, L^2]_{PB} = 0$ for all *i*) where $L^2 = \vec{L} \cdot \vec{L}$.
    c. Show that $[\vec{L}, f(r)]_{PB} = 0$ where *f* is an arbitrary function and $r = \sqrt{\vec{x} \cdot \vec{x}}$.

Note that parts b. &c. reflect a deeper result $[\vec{L}, s]_{PB} = 0$ for any scalar *s*. This reflects the fact that the angular momentum is the generator of rotations.

2. Label our canonical variables by a phase-space vector

$$\vec{\eta} = \begin{pmatrix} q_1 \\ q_2 \\ \cdots \\ q_n \\ p_1 \\ p_2 \\ \cdots \\ p_{n1} \end{pmatrix}$$ which satisfies canonical Poisson brackets $[\eta_i, \eta_j]_{PB} = J_{ij}$

now suppose that there is a one parameter continuous family of canonical transformations depending on one parameter ε : $\vec{\xi}(\vec{\eta};\varepsilon)$.

a. Show that $\vec{\xi}(\vec{\eta};\varepsilon)$ satisfies $[\xi_i, \xi_j]_{PB} = J_{ij}$ if it satisfies the conditions $\frac{d\xi_i}{d\varepsilon} = [\xi_i, g]_{PB}$ with $\boxed{\vec{\xi}(\vec{\eta};0) = \vec{\eta}}$ for some g. Note that this is the same form as usual Hamiltonian time with t replaced by ε and *H* replaced by *g*. The function *g* is called the generator of the transformation.

b. The time evolution of the phase-space position under Hamiltonian gives a transformation from an initial point in phase space to a subsequent one. That is $\vec{\eta}(t)$ is really a function the time and the initial conditions: $\vec{\eta}(\vec{\eta}_0,t)$. Note that the initial conditions are a set of phase space variable which are canonical. Now let me define a canonical transformation $\vec{\xi}(\vec{\eta},T)$ which has the same functional relation as $\vec{\eta}(\vec{\eta}_0,t)$. That is $\vec{\xi}(\vec{\eta},T)$ corresponds to the value the point in phase space that any point in phase space evolves into from $\vec{\eta}$ a time T later. Using part a) show that for any T, $\vec{\eta}(\vec{\xi},T)$ satisfies the canonical Poission bracket relation with H acting as the generator of time translations.