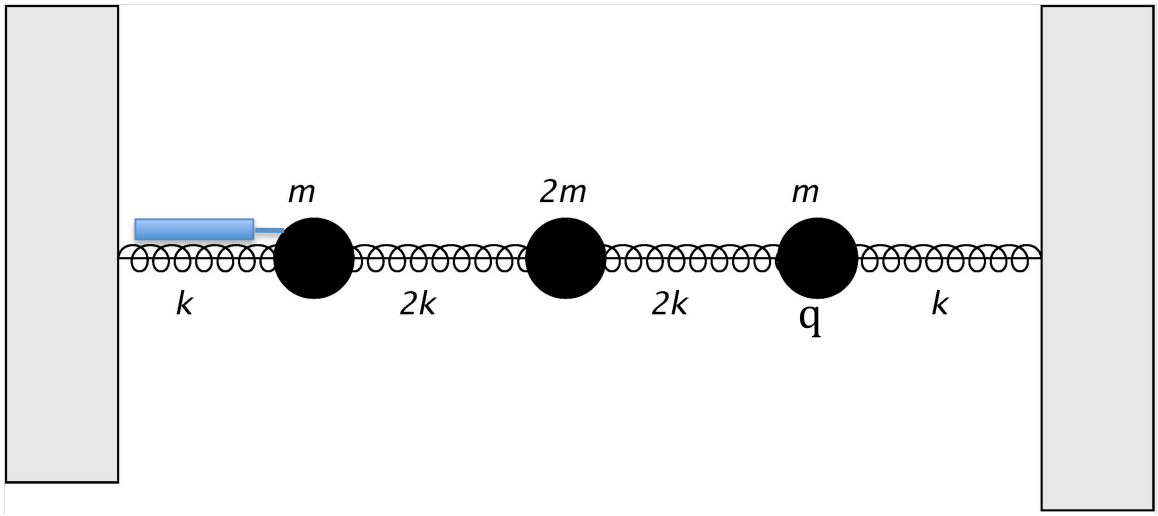
Physics 601 Homework 8---Due Friday October 30

1. Three beads are free to move along a wire. They are connected two immovable walls by 4 springs. The masses and spring constants are as indicated on the figure. The third particle is charged and has a mass q. The first particle is attached to a shock absorber with a force given by $F = -m\omega_0 \dot{x}$ where $\omega_0 = \sqrt{\frac{k}{m}}$. The last mass is charged. Suppose the system is place in an enectric field, oriented to the right with a magnitude of $E = -E_0 \cos(2\omega_0 t)$. Find the positions of each mass as a function of time, assuming the system has reached stady state.



2. Consider a two-body system with reduced mass μ and a potential of the form $V = -ar^{-k}$ for $a,k > 0$ (a,k real).
   a. Show that circular orbits exist for any $k \neq 2$ and find the relationship between the radius r_0 and L.
   b. Linearize the equation of motion for r around r_0 and
      i. Show that stable orbits only exist for $k < 2$
      ii. Find the oscillation frequency for fluctuations in r.
      iii. Find the values of k for which the orbits close.

3. Consider a two-body system with reduced mass μ and a potential of the form $V = ar^{k}$ for $a,k > 0$ (a,k real).
   a. Show that circular orbits exist for any k and find the relationship between the radius r_0 and L.
   b. Linearize the equation of motion for r around r_0 and
      i. Find the oscillation frequency for fluctuations in r.
      ii. Find the values of k for which the orbits close.

4. Use the equations of motion to show that for systems described by the Lagrangian $L = \frac{1}{2}\mu\dot{\vec{x}}^2 + \frac{\alpha}{r}$ (that is a Coulomb or gravitational system) show that
    a. The Runge-Lenz vector defined by $\vec{A} = \vec{p} \times \vec{L} - \alpha\mu\hat{r}$ (where $\hat{r} = \vec{x}/r$) is conserved.
    b. The Runge-Lenz vector is in the plane of the orbit