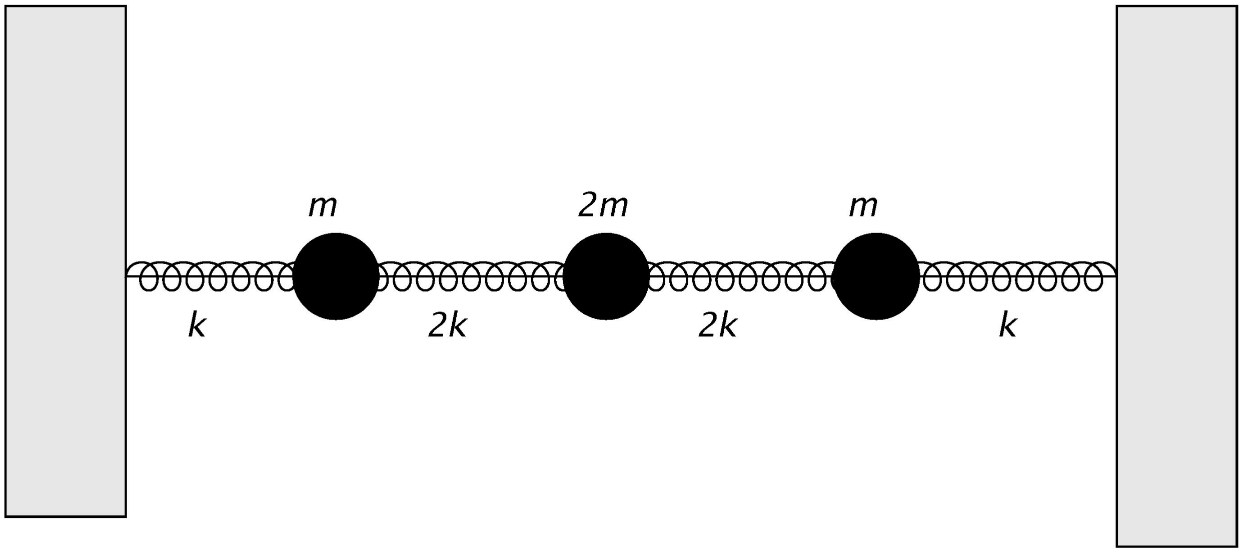
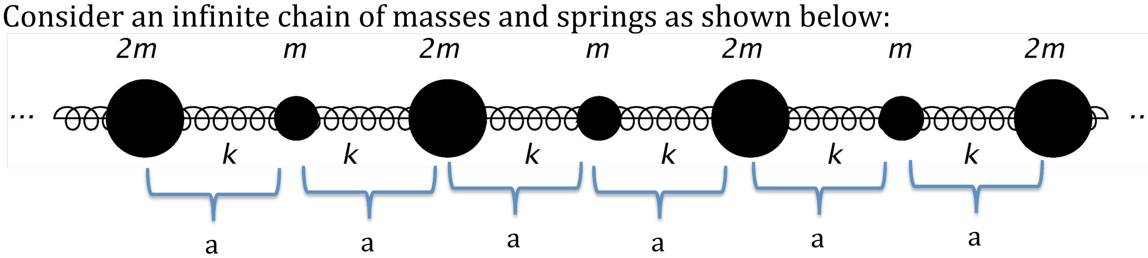
Physics 601 Homework 7---Due Friday October 23

1. Three beads are free to move along a wire. They are connected two immovable walls by 4 springs. The masses and spring constants are as indicated on the figure.



   a. Find the normal modes and their associated frequencies by finding the eigenvectors and eigenvalues of the appropriate matrix.
   b. Verify that the modes are orthogonal with respect to the mass matrix.
   c. Suppose at t=0 all three particles are in their equilibrium positions with the two particle on the end at rest and the one in the middle moving with velocity v. (This can happens as a result of an impulse acting on the middle particle). Find the motion of the three particles.

2. Consider an infinite chain of masses and springs as shown below:



   The masses alternate between two types, one twice as heavy as the other. The springs connecting the masses are all equal and in equilibrium the masses are all a distance *a* apart. The masses are constrained to move longitudinally. Find the dispersion relation relating frequency to wave number.

3. In class we found the Green's function for the harmonic oscillator. In this problem, I want you to find and use the analogous one for a damped oscillator. The damped

driven oscillator satisfied the equation: $m\ddot{x} + 2\beta m\dot{x} + m\omega_0^2 x = f(t)$ where β is a damping parameter. The solution is $x(t) = \int_{-\infty}^{\infty} dt' G(t,t') f(t')$ where the Green's function satisfies $(\partial_t^2 + 2\beta\partial_t + \omega_0^2)G(t,t') = \delta(t-t')$. A useful first step in constructing this is to exploit the known solution for steady state motion with a harmonic driving force: $(\partial_t^2 + 2\beta\partial_t + \omega_0^2)x(t) = f_0 e^{i\Omega t}$ has a solution of the form of the form $x(t) = \dfrac{f_0 e^{i\Omega(t-t')}}{\omega_0^2 - \Omega^2 + 2i\beta\Omega}$. Thus $(\partial_t^2 + 2\beta\partial_t + \omega_0^2)\dfrac{e^{i\Omega(t-t')}}{\omega_0^2 - \Omega^2 + 2i\beta\Omega} = e^{i\Omega(t-t')}$. Let us now integrate both sides with respect to Ω and divide by 2π:

$$\frac{\int_{-\infty}^{\infty} d\Omega\, (\partial_t^2 + 2\beta\partial_t + \omega_0^2)\dfrac{e^{i\Omega(t-t')}}{\omega_0^2 - \Omega^2 + 2i\beta\Omega}}{2\pi} = \frac{\int_{-\infty}^{\infty} d\Omega\, e^{i\Omega(t-t')}}{2\pi}$$

. We know that

$$\frac{\int_{-\infty}^{\infty} d\Omega\, e^{i\Omega(t-t')}}{2\pi} = \delta(t-t').$$

Moreover on the left hand side we can pull $(\partial_t^2 + 2\beta\partial_t + \omega_0^2)$ out of the integral as it does not depend on Ω. Thus

$$(\partial_t^2 + 2\beta\partial_t + \omega_0^2)\left(\frac{\int_{-\infty}^{\infty} d\Omega\, \dfrac{e^{i\Omega(t-t')}}{\omega_0^2 - \Omega^2 + 2i\beta\Omega}}{2\pi}\right) = \delta(t-t')$$

and the object in the parenthesis is the Green's function.

a. Evaluate the integral above using contour integration to find an explicit expression for $G(t,t')$. Note that the complex exponential implies that the ½ plane in which the contour is to be closed depends on the sign of *t-t'*.
b. Use this Green's function to find a solution of $m\ddot{x} + 2\beta m\dot{x} + m\omega_0^2 x = f_0 e^{-\Gamma t}\theta(t)$.
c. Consider the result in b. in the regime where Γ>>ω>>β. In that regime the system should look like an underdamped oscillator getting a delta-function-like impulse at t=0. Does it?