

Physics 601 Homework 13---Due Friday Dec. 11

1. Last week, I had you consider the case of a physical pendulum in the regime the high energy regime where the kinetic energy is much larger than the potential. This time I want to consider the opposite regime---extremely low energies. Again the Lagrangian is given by $L = \frac{1}{2}I\dot{\theta}^2 + V_0 \cos(\theta)$ where I is the moment of inertia and V_0 is the maximum value of the potential energy (i.e. $m g L$) where L is length from the pivot point to the center of mass. The problem under consideration is this: suppose that at $t=0$ the system is at rest with $\theta = \theta_i$. I wish to develop a systematic power counting scheme for this problem.

Consider the following power counting scheme in terms of λ : $L = \frac{1}{2}I\dot{\theta}^2 + \frac{V_0 \cos(\lambda^{1/2}\theta)}{\lambda}$.

- a. Assuming this system is at low energy and θ is small and one can expand out the cosine. Show that this is sensible in that leading order term with the harmonic term in leading order, a quartic term as a perturbation and a sextic term as a smaller perturbation and so on.
 - b. One can write $\theta = \theta_0 + \lambda\theta_1 + \lambda^2\theta_2 + \dots$. Find explicitly the form for θ_1 and θ_2 .
2. The approximate solution found in problem with naïve perturbation theory does not impose periodicity. However on general grounds we expect that $\theta(t) = \sum_n c_n \cos((2n+1)\omega t)$. Match this form onto the perturbative solution using the power counting scheme $\omega = \omega_0 + \lambda\omega_1 + \lambda^2\omega_2 + \dots$, to compute the frequency as a function of amplitude up to the order computed. Show that this gives $\omega(\theta_0)$ as a series in θ_0 with accurate terms up to 4th order.
 3. There is an alternative way to compute the frequency for the preceding problem---using energy conservation.

- a. Show that
$$\omega = \frac{\pi}{\int_{-\theta_i}^{\theta_i} \frac{d\theta \sqrt{I\lambda}}{\sqrt{2V_0(\cos(\lambda^{1/2}\theta_i) - \cos(\lambda^{1/2}\theta))}}}$$
.

- b. Expand this out to second order in the expansion and verify that it agrees with the result in part 2. Hint: It is easier to first expand and then integrate. Mathematica helps greatly in evaluating the integrals.

4. Consider a particle of mass m and charge q particle moving in a spatial constant (but time varying magnetic field).
- Using an argument analogous to that done in the case of the harmonic oscillator with varying spring constant, show that $\frac{v^2}{B}$ where v is the speed of the particle is an adiabatic invariant.
 - Note that in the adiabatic limit any given time the particle is making circular orbits (with the radius varying in time as the magnetic field strength changes) show that the magnetic flux threading through the circular orbit is an adiabatic invariant.
 - Suppose a particle makes a circular orbit with radius r_0 in a magnetic field B_0 . What is the radius if the magnetic field is slowly increased to $2 B_0$.