Physics 601 Homework 4---Due Friday October 2

Jose and Saletan problems 5.9, 5.10, 6.1, 6.7

In addition:

1. In order to get intuition about classical mechanics it is sometimes useful to think quantum mechanically. In this problem I want you to focus on the relationship between action-angle variables and quantum mechanics (in its semi-classical regime). For one-dimensional quantum mechanics in its semi-classical limit, the spacing of energy levels varies slowly from level to level. (That is if one defines $\Delta E_n \equiv E_{n+1} - E_n$ then $\Delta E_n \approx \Delta E_{n+1}$ for n>>1.) The semi-classical quantization condition is given by $\frac{1}{2\pi}\oint p\,dq = (n+\frac{1}{2})\hbar$. Classically, the angle variable is, by construction, cyclic and $\dot{\phi}$ is thus a constant. Show that in the semi-classical regime , $\dot{\phi}$ in the underlying classical system is related to ΔE in the quantum system through $\dot{\phi} = \frac{\Delta E}{\hbar}$.

2. Consider the canonical transformation for a system with one degree of freedom generated by: $F^2(q,P,t) = (q+\frac{1}{2}gt^2)(P - m\,g\,t) - \frac{P^2 t}{2m}$.
   a. Find the explicit form for the canonical transform.
   b. Verify that it satisfies the canonical Poisson bracket relations.

   For the remainder of this problem consider particle moving in one dimension in a constant gravitational field $L(q,\dot{q}) = \frac{1}{2}m\dot{q}^2 - mgq$.

   c. Use the fact that for any A, $\frac{dA}{dt} = \{A,H\} + \frac{\partial A}{\partial t}$ to show that $\frac{dQ}{dt} = 0$ and $\frac{dP}{dt} = 0$.
   d. Part c. implies that K the Hamiltonian associated with Q , P must be zero up to a possibly time dependent constant independent of Q , P. Show from the explicit form of $\frac{\partial F^2(q,P,t)}{\partial t}$ that this is in fact the case.
   e. What is the physical interpretation of Q , P.
   f. Show that $F^2(q,P,t)$ satisfies a Hamilton-Jacobi equation: $H\left(q, \frac{\partial F^2}{dq}\right) + \frac{\partial F^2}{dt} = 0$.
   g. Introduce the function the $f(t,Q,P) \equiv F^2\big(q(Q,P,t),P,t\big)$. Show that $\frac{\partial f(t,Q,P)}{\partial t} = L\big(q(Q,P,t), \dot{q}(Q,P,t)\big)$.