Physics 601 Homework 1---Due Friday September 11

Hint: For some of these problems it will be helpful to use Mathematica or some other symbolic manipulation program. If you make use of such a program please include the output with your homework solutions.

Jose and Slatan problems 2.11, 3.2

In addition:

1. Consider the Lagrangian for a simple 1-dimesnional harmonic oscillator: $L(x,\dot{x}) = \frac{1}{2} m\dot{x}^2 - \frac{1}{2} m\omega_0^2 x^2$ where ω_0 is a parameter.

    a. Find the equation of motion from Lagrange's equations.

    Consider a change of variables to the generalized coordinate $q = x^3$

    b. Using the result of a., find the equation of motion for *q*.
    c. Find the Lagrangian for *q* (*i.e.* find $L(q,\dot{q})$).
    d. Find the Lagrange's equation of motion for $L(q,\dot{q})$.
    e. How do the results of b. and d. compare? Why is this expected?

2. Again consider a simple 1-dimensional harmonic oscillator with $L(x,\dot{x}) = \frac{1}{2} m\dot{x}^2 - \frac{1}{2} m\omega_0^2 x^2$. Consider a family of trajectories with $x(t)$ subject to the boundary conditions $x(0) = 0,\ x(T) = l$. Suppose further that the family of trajectories includes the solution of the exact equations of motion. An example of such a family is the set of function $x(t,\omega) = \frac{l \sin(\omega t)}{\sin(\omega T)}$ where ω is a parameter.

    a. Verify that this family satisfies the boundary conditions.
    b. Show that for $\omega = \omega_0$ this trajectory correspond to the solution of the full equations of motion.
    c. Calculate the action as a function of ω: *i.e.* find $S(\omega)$.
    d. Show that $\frac{dS(\omega)}{d\omega} = 0$ at $\omega = \omega_0$. Explain why this is expected

3. Problem 2 considered a family of trajectories containing the exact solution of the equations of motion. Suppose one has a when a family of trajectories which does **not** contain the exact solution. The variational principal can still be of use in finding approximate solutions. Consider the harmonic oscillator

of problem 2 with the same boundary conditions. Consider the family of trajectories given by $x(t,c) = l\frac{t}{T} + c\,l\left(\frac{t^3}{T^3} - \frac{t}{T}\right)$ where *c* is a parameter.

a. Verify that this family satisfies the boundary conditions.
b. Calculate the action as a function of *c*: *i.e.* find $S(c)$.
c. Minimize $S(c)$ to find the "best" approximation to the full solution of the form considered.
d. Plot the exact solution and the approximate solutions for three cases: $i)\,T\omega_0 = 1 \quad ii)\,T\omega_0 = 2 \quad iii)\,T\omega_0 = 3$. Qualitatively discuss the difference in these cases and why these differences make sense.