Phys 601 by Dr. Agashe (Fall 2018)

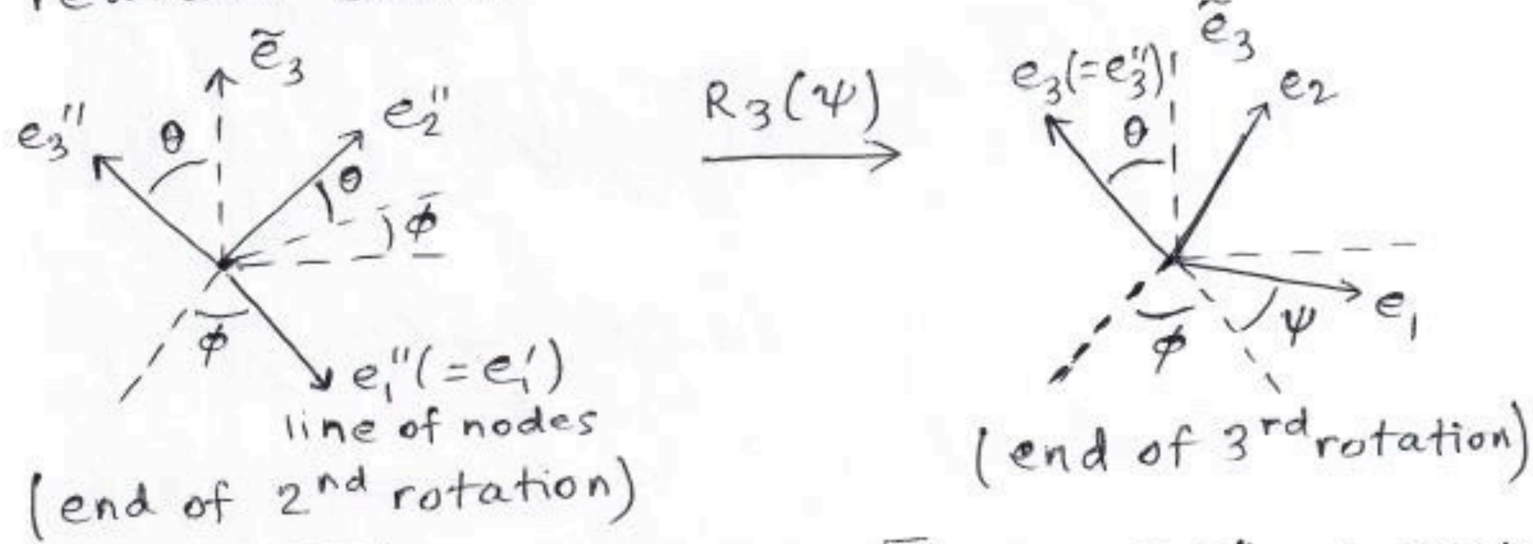
Expressing $\boxed{\tilde{\bar{e}}_3}$ (unit vector along z-axis of fixed/space frame) in terms of $\bar{e}_a$ (unit vectors of moving/body frame): see 1st line of Eq. 3.57 of DT's notes

$\boxed{\text{Method } 1}$: Use $\bar{e}_a = R_{ab}\tilde{\bar{e}}_b$ (rather its inverse), with R given in terms of Euler angles in Eq. 3.54 of DT's notes, i.e.,

$$\tilde{\bar{e}}_3 = (R^{-1})_{3a}\,\bar{e}_a = (R^T)_{3a}\,\bar{e}_a = R_{a3}\,\bar{e}_a$$

$$= \sin\theta\sin\psi\,\bar{e}_1 + \sin\theta\cos\psi\,\bar{e}_2 + \cos\theta\,\bar{e}_3$$

$\boxed{\text{Method } 2}$ Consider the before and after of 3rd rotation (by angle ψ about z-axis after 2nd rotation): see Fig. 39 of DT's notes, redrawn below



- End of $\boxed{2^{nd}}$ rotation gives: $\tilde{\bar{e}}_3 = \cos\theta\,\bar{e}_3'' + \sin\theta\,\bar{e}_2''$, since $\tilde{\bar{e}}_3 \perp \bar{e}_1''$ (line of nodes), i.e., has vanishing component along it

— 3^{rd} rotation (i.e., z-axis (in left picture) untouched; $\bar{e}_1''$ and $\bar{e}_2''$ rotated by ψ) gives

$\bar{e}_3'' = \bar{e}_3$; $\bar{e}_1 = \cos\psi \, \bar{e}_1'' + \sin\psi \, \bar{e}_2''$ and

$\bar{e}_2 = -\sin\psi \, \bar{e}_1'' + \cos\psi \, \bar{e}_2''$

or $\bar{e}_2'' = \sin\psi \, \bar{e}_1 + \cos\psi \, \bar{e}_2$

— Plugging above $\bar{e}_3''$ and $\bar{e}_2''$ into expression for $\bar{\tilde{e}}_3$ on previous page (bottom) gives

$$\bar{\tilde{e}}_3 = \cos\theta \, \bar{e}_3 + \sin\theta \sin\psi \, \bar{e}_1 + \sin\theta \cos\psi \, \bar{e}_2$$