(The following is a more detailed or explicit derivation of the solution given in Eq. 2.126 of DT's notes of the system in Eq. 2.124.)

We would like to solve:

$$\ddot{\boldsymbol{\eta}} \;=\; F\boldsymbol{\eta} \tag{1}$$

(as in Eq. 2.124 of DT's notes) where $\boldsymbol{\eta}$ is a column vector with n entries (i.e., a $n \times 1$ matrix). Also, F is a $n \times n$ matrix which is (in general) not symmetric, but has real eigenvalues (λ^2), given by

$$F\boldsymbol{\mu}_a \;=\; \lambda_a^2\, \boldsymbol{\mu}_a,\; a = 1, 2..., n \tag{2}$$

where $\boldsymbol{\mu}_a$ are the corresponding eigenvectors, taken to be column vectors with n entries.[1] Note that (given that F might not be symmetric) these eigenvectors need not be orthogonal. Using these n eigenvectors as the *columns*, we construct a $n \times n$ matrix (denoted by P) as follows:

$$P \;=\; \left(\boldsymbol{\mu}_1\; \boldsymbol{\mu}_2\; ...\; \boldsymbol{\mu}_n\right) \tag{3}$$

The matrix F can then be diagonalized using P and its inverse (which is *not* in general its transpose, since F need not be symmetric):

$$P^{-1}FP \;=\; \text{diag}\left(\lambda_1^2,\; \lambda_2^2, ...,\; \lambda_n^2\right) \tag{4}$$

[If you wish to check the above statement explicitly, then note that we can write P^{-1} in terms of its *rows* as:

$$P^{-1} \;=\; \begin{pmatrix} \boldsymbol{\zeta}_1^T \\ \boldsymbol{\zeta}_2^T \\ ... \\ \boldsymbol{\zeta}_n^T \end{pmatrix} \tag{5}$$

where $\boldsymbol{\zeta}_a$'s ($a = 1, 2, ..., n$) are *column* vectors with n entries. Then,

$$P^{-1}\, P \;=\; \text{identity} \tag{6}$$

along with the above forms of P and P^{-1} implies that

$$\boldsymbol{\zeta}_a^T \,.\, \boldsymbol{\mu}_b \;=\; \delta_{ab} \tag{7}$$

Plugging in above forms of P and P^{-1} into LHS of Eq. (4) and using Eqs. (2) and (7), we easily get the RHS of Eq. (4).

As somewhat of a detour, multiplying both sides of Eq. (4) from the right by P^{-1}, we get

$$P^{-1}F \;=\; \text{diag}\left(\lambda_1^2,\; \lambda_2^2, ...,\; \lambda_n^2\right)\; P^{-1} \tag{8}$$

[1] For obvious reasons, DT's notes refer to these as *right* eigenvectors.

Plugging in form of P^{-1} in Eq. (5) into above, we get

$$\boldsymbol{\zeta}_a^T F = \lambda_a^2 \boldsymbol{\zeta}_a^T \tag{9}$$

i.e., $\boldsymbol{\zeta}_a$'s can be called (as in DT's notes) *left* eigenvectors of the matrix F.]

Inserting the identity matrix in the form of PP^{-1} in-between F and $\boldsymbol{\eta}$ on the RHS of Eq. (1) and then multiplying by P^{-1} on both sides, we get

$$P^{-1}\ddot{\boldsymbol{\eta}} = P^{-1}F\left(PP^{-1}\right)\boldsymbol{\eta} \tag{10}$$

Using Eq. (4) for the factor at the beginning of RHS in above gives

$$(P^{-1}\ddot{\boldsymbol{\eta}}) = \text{diag}\left(\lambda_1^2,\ \lambda_2^2, ...,\ \lambda_n^2\right)\left(P^{-1}\boldsymbol{\eta}\right) \tag{11}$$

Because of the diagonalized form of the above system of equations, the solution is simply

$$P^{-1}\boldsymbol{\eta} = \left[\left(A_1 e^{+\lambda_1 t} + B_1 e^{-\lambda_1 t}\right),\ \left(A_2 e^{+\lambda_2 t} + B_2 e^{-\lambda_2 t}\right), ...,\ \left(A_n e^{+\lambda_n t} + B_n e^{-\lambda_n t}\right)\right]^T \tag{12}$$

where A_a and B_a are intergration constants. Multiplying by P from the left, we get

$$\boldsymbol{\eta} = P\left[\left(A_1 e^{+\lambda_1 t} + B_1 e^{-\lambda_1 t}\right),\ \left(A_2 e^{+\lambda_2 t} + B_2 e^{-\lambda_2 t}\right), ...,\ \left(A_n e^{+\lambda_n t} + B_n e^{-\lambda_n t}\right)\right]^T \tag{13}$$

Finally, plugging the form of P from Eq. (3) into RHS of the above gives

$$\boldsymbol{\eta} = \sum_a \boldsymbol{\mu}_a \left(A_a e^{+\lambda_a t} + B_a e^{-\lambda_a t}\right) \tag{14}$$

i.e., Eq. (2.126) of DT's notes. So, the *general* solution of Eq. (1) is a *superposition* of normal modes as in Eq. (14) above, where λ_a^2 is eigennvalue corresponding to eigenvector μ_a of F, as in Eq. (2).