

(Total 1 page, 3 problems, 150 points. Please write YOUR NAME on your exam book NOW!)

1. An electron  $e^-$  with kinetic energy  $K = 1$  MeV collides head-on with a positron  $e^+$  (positron has the same mass but opposite charge as the electron) at rest. After the collision the two particles annihilate into two gamma rays (very high frequency electromagnetic radiation). Find the energy  $E$ , momentum  $p$  and angle of emission  $\theta$  of the gamma rays. (40 points)

2. a. An FM transmitter operating at 100 MHz has a power output of 100kW. How many photons per second does the transmitter emit? (15 points)

b. From the Bohr postulate on the quantization of angular momentum derive an expression for the radius of the first Bohr orbit in  $He^+$  in terms of the electron mass  $m_e$ , charge  $e$  and  $k$ , the Coulomb constant. (30 points)

c. An atom in an excited state 1 eV above the ground state remains in that state  $1\mu$  sec before transiting to the ground state. What is the uncertainty in energy? (15 points)

3. The wave function of a quantum oscillator with mass  $m$  and natural frequency  $\omega$  in its ground state is given by

$$\psi_0(x) = (a/\pi)^{\frac{1}{4}} e^{-ax^2}$$

where  $a = \frac{m\omega}{\hbar}$ .

a. Calculate  $\Delta x \equiv \sqrt{x^2 - \langle x \rangle^2}$  [Use symmetry arguments if possible, but give the reasoning clearly. The following formula may prove useful:

$$\int_0^{\infty} x^2 e^{-ax^2} dx = \frac{1}{4a} \sqrt{\frac{\pi}{a}}$$

(20 points)

b. Calculate  $\Delta p \equiv \sqrt{p^2 - \langle p \rangle^2}$  [Energy relation can be invoked to relate  $\langle p^2 \rangle$  to  $\langle x^2 \rangle$ ] (20 points)

c. Evaluate  $\Delta x \Delta p$ . Does this wave function obey the uncertainty principle? (10 points)

**Note:** Please show your derivations and reasoning. You won't get any credit if you just lift the expression from your formula sheet for your answer.

Some useful constants and formulas: Planck's constant  $h = 6.63 \times 10^{-34} Js$ ,

electron's rest mass = 0.511MeV,  $1 \text{ eV} = 1.6 \times 10^{-19} J$ .

For numbers, kilo= $10^3$ , Mega= $10^6$ , mili= $10^{-3}$ ,  $\mu = 10^{-6}$