

(Total 2 pages, 7 problems, 250 points)

—Remember to write down YOUR NAME on each of your exam books—

1. (40 points) Consider a particle with energy E approaching from the left ($x = -\infty$) a step potential with height U at $x = 0$ (i.e., $U = 0$ for $x < 0$ and $= U$ for $x > 0$). Denote the wave numbers of the incident and transmitted waves by k_1, k_2 respectively.

A. Write down the form of the wave function on both regions and the junction conditions (12 points)

B. Derive (not just quote) an expression for the reflection coefficient R and the transmission coefficient T in terms of k_1, k_2 (12 points)

C. Calculate the fraction of 25 MeV protons reflected and the fraction transmitted by a 20 MeV step. Do the same for an electron (16 points).

2A (15 points) Calculate the most probably distance of the electron in the ground state of hydrogen from the nucleus. The wave function is $\psi_{100} = (\pi a_0^3)^{-1/2} e^{-r/a_0}$.

2B. (20 points) Calculate the probability that the hydrogen 1s electron will be found inside the first Bohr radius a_0 .

3A. (20 points) Draw an energy diagram for a hydrogen electron in the 2s state and 1s state when the atom is immersed in a magnetic field B , assuming the electron is spinless. Draw another diagram when electron spin is included. Label the states clearly with the quantum numbers m_l, m_s . Express the energy spacing ΔE in terms the Bohr magneton $\mu_B = \frac{e\hbar}{2m_e}$.

3B. (10 points) How does the spin-orbit (LS) coupling alter the 2p level structure of hydrogen? Draw an energy level diagram with and without LS coupling. Label the levels in spectroscopic nl_j notations.

4. (35 points) A. Explain the physical origin of exchange force for an atom with two electrons. [Reason in steps how the Pauli Principle, the spin of the electron, the symmetry of the total, space, and spin wave function enter into the picture.] (15 points)

B. Sketch the energy diagrams for the ground state (1s 1s) and excited states (1s 2s) and (1s 2p) of the helium atom, a) with Coulomb force and (5 points) b) with Coulomb and exchange force (10 points). Label the singlet and triplet states. Which state has the lower energy? Why? (5 points)

5 (20 points) A. Write down the electronic configuration of ^{13}Al atom. (8 points)

B. What is Hund's rule and its origin? (5 point)

C. Which electronic configuration has a lower energy $[\text{Ar}]3d^44s^2$ or $[\text{Ar}]3d^54s^1$? Why? (7 points) (The notation $[\text{Ar}]$ represents the filled configuration of Argon)

6A. (20 points) Derive an expression for the density of states $\mathcal{D}(\epsilon)$ of N non-relativistic quantum particles of mass m and energy ϵ in a cubic box of length L .

6B. (20 points) For phonons, integrating the density of state function up to the Debye frequency ν_D gives $3N$, the number of modes per mole. Derive an expression for ν_D in terms of N and $V = L^3$. (12 points) What is the physical meaning of Debye temperature T_D ? (4 points) At what temperature range (compared to T_D) will the heat capacity at constant volume C_V for an ideal gas approach $3R$? (4 points)

6C (20 points) Given that the Fermi energy of aluminum is 11.63 eV, determine its valence. a) First, calculate the number of free electrons per unit volume at low temperatures. (12 points) b) Second, calculate the number of aluminum atoms per unit volume from its density 2.7g/cm^3 and atomic weight 27. (8 points)

7. (30 points) a) How does the band structure of a solid arise? Describe how one can distinguish a solid to be a conductor, an insulator or a semiconductor (name two essential factors) b) What is a Brillouin zone? What is the effective mass m^* of an electron in a periodic lattice? c) What is the distinction between intrinsic and extrinsic semiconductors? d) Describe some key features of the BCS theory of superconductivity.

Note: Please show your derivations and reasoning. You won't get any credit if you just lift the expression from your formula sheet for your answer.

Some useful constants and formulas: Planck's constant $h = 6.63 \times 10^{-34} \text{Js}$

Electron mass $m_e = 9.11 \times 10^{-31} \text{kg}$; $1 \text{eV} = 1.6 \times 10^{-19} \text{J}$; $g = 9.8 \text{m/s}^2$. Avagadro's number $N_A = 6.02 \times 10^{23} \text{atoms/mole}$.