Problems

By the end of this homework, chances are, you will be tired of computing the numerical value for the product hc. Here's a little suggestion-- these problems are simplified if we express the combination of hc in units of $eV \cdot nm$. That way you can work in convenient units of eV and nm: $hc = 1.986 \times 10^{-25} J \cdot m = 1239.7 eV \cdot nm$.

1. In the typical experimental photoelectric set-up, a Geissler tube (two electrodes in a evacuated glass tube) is connected to some voltage source. In this context, state the difference between the *stopping potential*, a *retarding* voltage and an *accelerating* voltage. Feel free to draw diagrams/graphs to make your point clear.

When a piece of metal is irradiated with UV light, electrons are ejected from the surface. If the metal is connected to one end of a potential difference, the electrons will see an electric field and feel an electric force. The electric field lines are oriented so that they start on the positive plate and end on the negative plat; in other words the electric force on the electron is always directed towards the positive plate.

Accelerating voltage: If the voltage is set up so that the photoelectrons (i.e. those ejected from the action of the UV light) come off the negative plate, they will feel an accelerating voltage towards the positive plate.



Retarding voltage: If the voltage is reversed so that the photoelectrons come off the positive plate the electrons will feel a decelerating voltage towards the negative plate. The electrons will have a range of velocities (KE) when they are ejected from the surface so, in general, slow electrons will not make it to the negative plate but fast ones will.



Stopping potential: If we increase the retarding voltage so that not even the fastest electrons make it, then the photocurrent stops. This is called the stopping potential. Even those electrons emitted with the highest speeds just fail to reach the plates.

2. State and clearly explain three *classical* (i.e. Maxwell model for light) predictions regarding the photoelectric effect. Feel free to draw diagrams/graphs to make your point clear.

(1) The maximum kinetic energy should be proportional to the intensity of the radiation.

(2) The photoelectric effect should occur for light of any frequency.
(3) There's a measurable time delay between the radiation first striking the surface and the first electrons being emitted.
(For a full detail explanation of each one, please refer to the book)

3. A laser beam with an intensity of 120 W/m² (roughly that of a small helium-neon laser) is incident on a surface of sodium. It takes a minimum energy of 2.3 eV to release an electron from sodium (the work function Φ of sodium). Assuming the electron to be confined to an area of radius equal to that of a sodium atom (0.10 nm), how long will it take, *classically*, for the surface to absorb enough energy to release an electron?

Classically, the average power P_{av} delivered by the waves of intensity I to an area A is IA. An atom on the surface displays a 'target area' of $A = \pi r^2 = \pi (0.10x10^{-9} m)^2 = 3.1x10^{-20} m^2$. If the entire electromagnetic power is delivered to the electron, energy is absorbed at the rate $\Delta E/\Delta t = P_{av}$. The time interval Δt necessary to absorb an energy $\Delta E = \Phi$ can be expressed as

$$\Delta t = \frac{\Delta E}{P_{av}} = \frac{\Phi}{IA} = \frac{(2.3eV)(1.6x10^{-19} J/eV)}{(120W/m^2)(3.1x10^{-20}m^2)} = 0.10 s$$

- 4. State and clearly explain three *experimental* facts about the photoelectric effect. Feel free to draw diagrams/graphs to make your point clear.
 - (1) The maximum kinetic energy (determined from the stopping potential) is totally independent of the intensity of the light source.
 - (2) The maximum kinetic energy is a function of the frequency of light.

(3) The photoelectric effect does not occur at all if the frequency of the light source is below a certain value.

(4) The first photoelectrons are emitted virtually instantaneously (within 10^{-9} s) after the light source is turned on. (For a full detail explanation of each one, please refer to the book)

5. (a) What are the energy and momentum of a photon of red light of wavelength 650 nm? (b) What is the wavelength of a photon of energy 2.40 eV? *Recall from relativity that for a massless particle,* E = pc.

(a)
$$E = hf = \frac{hc}{\lambda} = \frac{(6.63x10^{-34} J \cdot s)(3.00x10^8 m/s)}{650x10^{-9} m} = 3.06x10^{-19} J = 1.91 eV.$$

(b) $p = \frac{E}{c} = \frac{h}{\lambda} = \frac{(6.63x10^{-34} J \cdot s)}{650x10^{-9} m} = 1.02x10^{-27} kg \cdot m/s = 1.91 eV/c.$

6. The work function for tungsten metal is 4.52 eV. (a) What is the cutoff wavelength λ_c for tungsten? (b) What is the maximum kinetic energy of the electrons when radiation of wavelength 198 nm is used? (c) What is the stopping potential in this case?

(a) Starting from Einstein's photoelectric equation $KE_{max} = hf - \phi$, the cutoff frequency is when the electron has received just enough energy to overcome the binding energy (a.k.a. the work function). Mathematically,

$$0 = hf_c - \phi = \frac{hc}{\lambda_c} - \phi$$
$$\lambda_c = \frac{hc}{\phi} = \frac{(6.63x10^{-34} J \cdot s)(3x10^8 m/s)}{(4.52 eV)(1.60x10^{-19} J/eV)} = 275 \text{ nm (in the UV)}$$

(b) The maximum kinetic energy is just given by,

$$KE_{\max} = hf - \phi = \frac{hc}{\lambda} - \phi$$

= $\frac{(6.63x10^{-34} J \cdot s)(3x10^8 m/s)}{198x10^{-9} m} - 4.52 eV(1.602x10^{-19} J/eV) = 1.75 eV.$

(c) The stopping potential is the voltage corresponding to KE_{max} :

$$V_{stopping} = \frac{KE_{max}}{e} = \frac{1.75 \, eV}{e} = 1.75 \, Volts$$

7. *Extra credit problem 1*. **SMM, Chapter 2, Problem 19.**

We can solve this problem numerically from the very beginning, but we'll do it analytically first and then plug in at the last moment.

For
$$\lambda_1$$
, $KE_{1 \max} = 1.00 \text{ eV}$, we have $KE_{1 \max} = \frac{hc}{\lambda_1} - \phi$.
For $\lambda_2 = \lambda_1/2$, $KE_{2 \max} = 4.00 \text{ eV}$, we have $KE_{2 \max} = \frac{hc}{(\lambda_1/2)} - \phi$.

Solving one of them for the work function and substituting into the other one, we get

$$KE_{2\max} - KE_{1\max} = \frac{hc}{\lambda_1}$$

We can substitute this expression for hc/λ_1 in the first equation and solve for the work function.

$$\phi = KE_{2 \max} - 2KE_{1 \max} = 4.00 \ eV - 2.00 \ eV = 2.00 \ eV$$

8. *Extra credit problem 2.* Consider a photon with wavelength λ incident on an unknown metal. The most energetic electrons ejected from the metal are bent into a circular orbit of radius *R* by a magnetic field whose strength is equal to *B*. (a) Derive an expression for the work function in terms of these variables. (b) Use this expression to solve **SMM, Chapter 2, Problem 21.**

(a) A charged particle, with charge q and velocity v, traveling perpendicular to a uniform magnetic field B will trace out a circular arc of R where the magnetic

force acts like a centripetal force: $F_B = qvB = m\frac{v^2}{R}$. Solving for the velocity,

$$v = \frac{qBR}{m}$$

Using Einstein's photoelectric equation and solving for the work function,

$$\phi = \frac{hc}{\lambda} - \frac{1}{2}mv^2$$

 $\phi = hf - KE$

Substituting for the velocity in terms of B, R, q and m

$$\phi = \frac{hc}{\lambda} - \frac{1}{2}m\left(\frac{qBR}{m}\right)^2$$

Simplifying and realizing that we are using electrons,

$$\phi = \frac{hc}{\lambda} - \frac{e^2 B^2 R^2}{2m_e}$$

(b) We can now use this equation to do SMM, Chapter 2, Problem 21. Plugging all the parameters as given,

$$\phi = \frac{1239.7 \, eV \cdot nm}{450 \, nm} - \frac{(1.602 x 10^{-19} \, C)^2 (2.0 x 10^{-5} \, T)^2 (0.20 m)^2}{2(9.11 x 10^{-31} \, kg)} \frac{1}{1.602 x 10^{-19} \, J \, / \, eV}$$

 $\phi = 2.755 \, eV - 1.407 \, eV = 1.348 \, eV.$