PHYS420 (Spring 2002) **CLASS ID NUMBER___________**
Riq Parra

Exam #1
(Friday, March 1st, 2002)

Problem 1 Short answer problem (35 points total – 5 points each)

(1) What was the ether?

The ether was thought to be the required medium, present everywhere, even in empty space, in which electromagnetic radiation (including visible light waves) propagated.

(2) Define an inertial reference frame.

An inertial reference is a system in which a free body is not accelerating.

(3) State Einstein's Postulates of Special Relativity.

(1) All the laws of physics are the same in all inertial frame.
(2) The speed of light in vacuum is constant in all inertial frames and has the value of $3.00x10^8$ m/s.

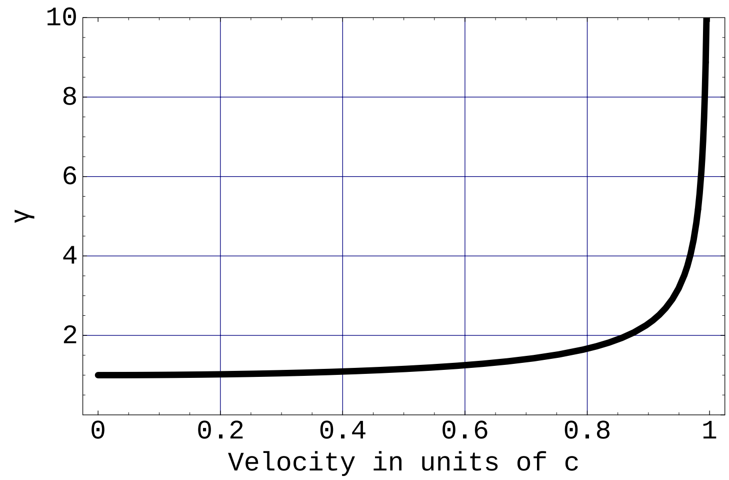
(4) What is a **proper** time interval?

A proper time interval is a time interval between two events as measured by an observer at rest with the events.

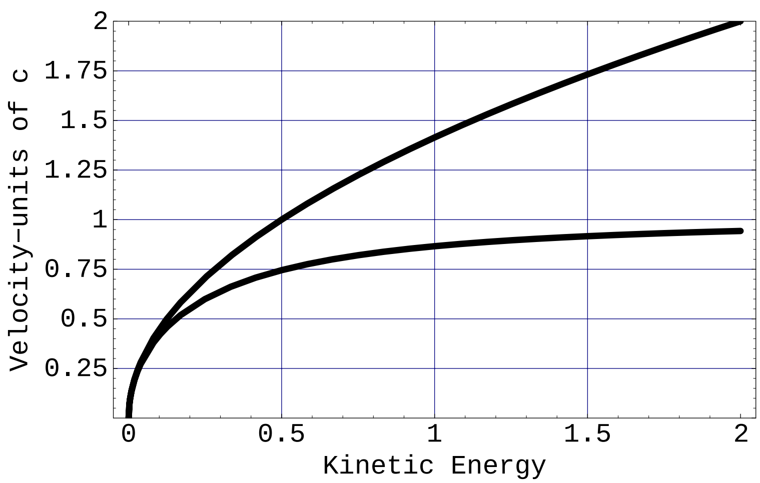
(5) A light source is known to emit light with a wavelength of 122 nm while at rest (in the ultraviolet region). While in motion, we observe the light to be Doppler shifted to 366 nm. Is the source approaching us or receding from us. Justify your answer.

If the source is approaching the observer, the observed wavelength gets shorter (i.e. $\lambda_{observer} < \lambda_{source}$). On the other hand, if the source is receding from the observer, the observed wavelength gets longer (i.e. $\lambda_{observer} > \lambda_{source}$). In our example, since ($\lambda_{observer} = 366$) > ($\lambda_{source} = 122$ nm), we conclude that the source is receding.

(6) Sketch a graph of the Lorentz factor (γ) versus velocity (in units of c).



(7) In particle accelerators, electrons are accelerated to very high speeds. In a single graph, sketch the acquired electron velocity (in units of *c*) versus (a) the relativistic kinetic energy and (b) the non-relativistic kinetic energy.



The top curve, which exceeds c, is the non-relativistic case. In the bottom curve, which is the relativistic case, we see that the velocity is always less than c.

PHYS420 (Spring 2002) **CLASS ID NUMBER___________**
Riq Parra

Exam #1
(Friday, March 1st, 2002)

Problem 2 (40 points)

Alpha Centauri, a nearby star in our galaxy, is 4.3 light-years away. This means that, as measured by a person on Earth, it would take light 4.3 years to reach this star. A rocket leaves for Alpha Centauri at a speed of v = 0.95c relative to the Earth. Assume that the Earth and Alpha Centauri are stationary with respect to one another.

(a) Work out the distance to Alpha C., as measured by Earth observers, in km? *(5 points)*

First we recall that,

$$1\,year = 1\,year\left(\frac{365\,days}{1\,year}\right)\left(\frac{24\,hrs}{1\,day}\right)\left(\frac{60\min}{1\,hr}\right)\left(\frac{60\sec}{1\min}\right) = 3.1536x10^{7}\sec.$$

Using the meaning of a light year (which is stated in the problem) then

$$d = c\,t = (3x10^{8}\ m/s)(4.3\,years) = \boldsymbol{4.07x10^{13}\ km}$$

(b) The distance calculated in (a), is it a proper distance? Justify your answer. *(5 points)*

Yes. According to the problem, this distance is 'as measured by a person on Earth'. Since Alpha Centauri is stationary with respect to the Earth, it is in the same inertial reference frame. The distance measurement is done by an observer at rest relative to the object being measured. Therefore, it's a proper distance, d_0.

(c) According to the astronauts, how much did they age (in years) during their journey? *(10 points)*

To an observer on Earth, it takes light 4.3 years to get to Alpha Centauri. It will take the astronauts traveling at v = 0.95c, t = (4.3 ly) /(0.95 c) =4.5 years. So the Earth observer thinks that the astronauts have aged 4.5 years. Since the Earth observer is not at rest with respect to the astronauts, this ***is not*** *a proper age measurement. On the other hand, the astronauts' measurement of their own aging* ***is*** *a proper time interval Δt_0. Using the time dilation formula and solving for t_0...*

$$\Delta t_0 = \Delta t\sqrt{1-\frac{v^2}{c^2}} = (4.5\,years)\sqrt{1-\left(\frac{0.95c}{c}\right)^2} = \boldsymbol{1.4\ years}$$

(d) According to the astronauts, how far (in km) did they travel? *(10 points)*

Since the spaceship is moving relative to the Earth – Alpha Centauri inertial reference frame, their determination of the distance traveled ***is not*** *a proper distance. Since at this point we have d_0, Δt and Δt_0, we can do the problem in two ways.*

(1) Use the length contraction formula directly,

$$d = \frac{1}{\gamma} d_0 = d_0 \sqrt{1 - \frac{v^2}{c^2}} = (4.07x10^{13}\ km)\sqrt{1 - \left(\frac{0.95c}{c}\right)^2} = \boldsymbol{1.27x10^{13}\ km}$$

(2) In the spaceship, the astronauts think they've traveled distance d in time t_0 with a velocity v (i.e. $v = \frac{d}{t_0}$). And so,

$$d = vt_0 = 0.95c(1.4\ years) = \boldsymbol{1.27x10^{13}\ km}$$

(e) One of the astronauts, using a meter stick, measures the length and diameter of the cylindrical spacecraft to be 82 and 21 m, respectively. Assuming that the spacecraft is oriented with its long cylindrical axis in the direction of motion, what are the dimensions of the spacecraft, as measured by an observer on earth? *(10 points)*

The length of 82 m is a proper length L_0, since it is measured using a meter stick that is at rest relative to the spacecraft. The length L measured by the observer on earth can be determined from the length-contraction formula. On the other hand, the diameter of the spacecraft is perpendicular to the motion, so the earth observer does not measure any change in the diameter.

$$\text{Ship's length} = L = \frac{1}{\gamma} L_0 = L_0 \sqrt{1 - \frac{v^2}{c^2}} = (82\ m)\sqrt{1 - \left(\frac{0.95c}{c}\right)^2} = \boldsymbol{26\ m}$$

Ship's diameter = ***21 m***

Problem 3 (20 points)

A 20-year-old political prisoner named Bob is exiled to travel space for 35 Earth years. By a strange twist of fate, his daughter is born the very day that he leaves to serve his sentence. How fast would he need to travel so that, when he arrives back on Earth, he's the same age as his daughter?

You have to love these silly contrived physics problems. Is this even possible? Let's go through the steps.
(1) The round trip must take 35 Earth years, so Δt *= 35 years (not a* ***proper*** *measurement of Bob's age).*
(2) Bob, at the time of departure, is 20 years old. Let's call this initial age τ *(= 20 yrs).*
(3) During the exile, Bob will have aged Δt_0 *years (a* ***proper*** *measurement of Bob's age) which is given by* $\Delta t_0 = \dfrac{\Delta t}{\gamma}$.
(4) At the time of his return, Bob's age will be equal to his initial age plus the time he aged during the trip. On the other hand, at the time of his arrival, his daughter will be 35 years old (which, of course is just, Δt*). Therefore putting everything together,*

$$(\text{Final Age})_{Bob} = (\text{Final Age})_{daughter}$$

$$\tau + \Delta t_0 = \Delta t$$

$$\tau + \frac{\Delta t}{\gamma} = \Delta t$$

Solving for $1/\gamma$,

$$\frac{1}{\gamma} = \frac{\Delta t - \tau}{\Delta t} = \sqrt{1 - \frac{v^2}{c^2}}$$

Solving for v and plugging in the numbers,

$$v = c\sqrt{1 - \left(\frac{\Delta t - \tau}{\Delta t}\right)^2} = c\sqrt{1 - \left(\frac{35\,years - 20\,years}{35\,years}\right)^2} = \boldsymbol{0.9035c}$$

PHYS420 (Spring 2002) **CLASS ID NUMBER___________**
Riq Parra

Exam #1
(Friday, March 1st, 2002)

Problem 4 (20 points)

A physics professor claims in court that the reason he went through the red light (λ = 650 nm) was that, due to his motion, the red color was Doppler shifted to green (λ= 550 nm). How fast was he going? *Useful reminder: The relation between frequency, f, and wavelength, λ, is given by* $c = \lambda f$.

This is a direct application of the Doppler formula. It's clear from the content of the problem that the driver is approaching the source. Therefore we use the formula,

$$f_{observer} = f_{source}\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$$

In terms of wavelength,

$$\frac{c}{\lambda_{observer}} = \frac{c}{\lambda_{source}}\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$$

Solving for v (there's a little algebra here) and plugging in we get,

$$v = \frac{\lambda_s^2 - \lambda_o^2}{\lambda_s^2 + \lambda_o^2}c = \frac{650^2 - 550^2}{650^2 + 550^2}c = \boldsymbol{0.1655c}$$

PHYS420 (Spring 2002) **CLASS ID NUMBER___________**
Riq Parra

Exam #1
(Friday, March 1st, 2002)

Problem 5 (20 points)

The particle accelerator at Stanford University is three kilometers long and accelerates electrons ($m_{electron} = 9.11x10^{-31}\,kg$) to a speed of 0.999 999 999 7 c, which is very nearly equal to the speed of light.
(a) What is the rest energy (in **MeV**) of the electrons? *(1 eV = 1.602x10^{-19} Joules)*
(b) Find the magnitude of the relativistic momentum of the electrons. Comparing it with the nonrelativistic value, is it bigger, smaller? By what factor?
(c) Compute the electron's total energy (in **GeV**).

(a) The rest energy of the electron is simply

$$E_{rest} = m_{electron}c^2 = (9.11x10^{-31})c^2 = 8.199x10^{-14}\ \text{Joules} = \mathbf{0.511MeV}$$

(b) We simply apply the relativistic momentum equation,

$$p_{relativistic} = \frac{mu}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{(9.11x10^{-31})(0.9999999997c)}{\sqrt{1-\frac{(0.9999999997c)^2}{c^2}}} = \mathbf{1.12x10^{-17} kg\ m/s}$$

The ratio of relativistic to non-relativistic momentum is just,

$$\frac{p_{relativistic}}{p_{classical}} = \frac{\cancel{mu}\Big/\sqrt{1-\frac{u^2}{c^2}}}{\cancel{mu}} = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.9999999997c)^2}{c^2}}} = 4x10^4$$

*So the relativistic momentum is **bigger by a factor of 40000***

(c) The electron's total energy is given by

$$E_{total} = \sqrt{p^2c^2 + (m_{electron}c^2)^2} = \sqrt{(1.12x10^{-17})^2c^2 + (8.199x10^{-14})^2} = 3.36x10^{-9}\ J$$

$$= \mathbf{20.97\ GeV}$$

PHYS420 (Spring 2002)
Riq Parra

Equation Sheet for Exam #1
(Friday, March 1st, 2002)

Lorentz Transformations

$$\left\{\begin{array}{l} x' = \gamma(x - vt) \\ y' = y \\ z' = z \\ t' = \gamma(t - \frac{v}{c^2}x) \end{array}\right\} \text{ where } \gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Velocity Transformation

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}, \quad u'_y = \frac{1}{\gamma}\frac{u_y}{1 - \frac{u_x v}{c^2}}, \quad u'_z = \frac{1}{\gamma}\frac{u_z}{1 - \frac{u_x v}{c^2}}$$

Time Dilation

$$t = \gamma t_0 = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Length Contraction

$$L = \frac{L_0}{\gamma} = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

Doppler Shift

$$f_{observer} = \frac{\sqrt{1 + (v/c)}}{\sqrt{1 - (v/c)}} f_{source}, \textit{ approaching source}$$

Momentum

$$p \equiv \frac{mu}{\sqrt{1 - \frac{u^2}{c^2}}}$$

Energy

$$E_{total} = \frac{mc^2}{\sqrt{1 - \frac{u^2}{c^2}}} = KE + E_{rest}; \qquad E_{rest} = mc^2$$

$$KE = \frac{mc^2}{\sqrt{1 - \frac{u^2}{c^2}}} - mc^2 = E_{total} - E_{rest}; \qquad E_{total}^2 = p^2c^2 + (mc^2)^2$$