PHYS 411 (Fall 2013): Electricity and Magnetism
Summary of topics/formulae for final exam

Chapter 2 of Griffiths

1. Coulomb's law: $\mathbf{F} = Q\mathbf{E}$; Electric field, $\mathbf{E}$ due to point charge $= \frac{q}{4\pi\epsilon_0}\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$
2. Gauss's law: $\nabla.\mathbf{E} = \frac{\rho}{\epsilon_0}$; $\int \mathbf{E}.d\mathbf{a} = \frac{Q_{enc}}{\epsilon_0}$
3. Potential: $\mathbf{E} = -\nabla V$; $V(\bar{r}) = -\int_{\mathcal{O}}^{\bar{r}} \mathbf{E}.d\mathbf{l}$; V due to point charge $= \frac{q}{4\pi\epsilon_0}\frac{1}{\mathscr{r}}$
4. Boundary conditions: $\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{n}}$; $V_{\text{above}} = V_{\text{below}}$
5. Work done to assemble point charges: $W = \frac{1}{4\pi\epsilon_0}\sum_{i\neq j}\frac{q_i q_j}{\mathscr{r}_{ij}}$
   Energy stored in electric field: $W = \frac{1}{2}\epsilon_0 \int E^2 d\tau$
6. Conductors: only surface charge; $\mathbf{E} = 0$ inside; $\mathbf{E} = \frac{\sigma}{\epsilon_0}$ at surface (and is normal to surface); entire conductor has same potential
7. Capacitance: $Q = CV$
   Parallel plate capacitor: $C = \frac{\epsilon_0 A}{d}$
   Energy stored in capacitor: $W = \frac{1}{2}CV^2$

Chapter 3 of Griffiths

1. Method of images for a conducting plane: value of image charge, $q' = -q$; location $-d$
   Method of images for a conducting sphere: value of image charge, $q' = -\frac{R}{a}$; location $b = \frac{R^2}{a}$
2. Cartesian separation of variables: solutions for each part are exponential or sinusoidal functions
   Spherical separation of variables: solutions for angular part are Legendre polynomials
3. Potential due to (pure) dipole at origin, aligned along z-axis: $V_{\text{dip}} = \frac{\mathbf{p}.\hat{\mathbf{r}}}{4\pi\epsilon_0 r^2}$

Chapter 4 of Griffiths

1. Torque on dipole: $\mathbf{N} = \mathbf{p} \times \mathbf{E}$
2. Bound charges: $\sigma_b = \mathbf{P}.\hat{\mathbf{n}}$; $\rho_b = -\nabla.\mathbf{P}$
3. Electric displacement: $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$; $\nabla.\mathbf{D} = \rho_f$; $\int \mathbf{D}.d\mathbf{a} = Q_{f_{enc}}$
4. Linear dielectric: $\mathbf{P} = \epsilon_0\chi_e\mathbf{E}$; $\epsilon_r = 1 + \chi_e$; $\mathbf{D} = \epsilon_0\epsilon_r\mathbf{E}$

Chapter 5 of Griffiths

1. Lorentz force law: $\mathbf{F}_{\text{mag}} = Q(\mathbf{v} \times \mathbf{B})$ (point charge); $\mathbf{F}_{\text{mag}} = I\int d\mathbf{l} \times \mathbf{B}$ (line current)

2. Continuity equation: $\nabla.\mathbf{J} = -\frac{\partial\rho}{\partial t}$

3. Biot-Savart law: magnetic field, $\mathbf{B} = \frac{\mu_0 I}{4\pi}\int\frac{d\mathbf{l}'\times\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}$

4. Ampere's law: $\int\mathbf{B}.d\mathbf{l} = \mu_o I_{enc} = \int\mathbf{J}.d\mathbf{a}$; $\nabla\times\mathbf{B} = \mu_0\mathbf{J}$

5. Magnetic vector potential (general): $\nabla.\mathbf{B} = 0 \Rightarrow \mathbf{B} = \nabla\times\mathbf{A}$

   Magnetic vector potential if current vanishes at infinity: $\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{J(r')}}{\mathscr{r}}d\tau'$

6. Boundary conditions: $\mathbf{B}_{\text{above}} - \mathbf{B}_{\text{below}} = \mu_0(\mathbf{K}\times\hat{\mathbf{n}})$

7. Magnetic dipole (general): $\mathbf{A}_{\text{dip}}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{\mathbf{r}}}{r^2}$, with $\mathbf{m} = I\int d\mathbf{a}$

   Magnetic dipole at origin pointing along z-direction: $\mathbf{A}_{\text{dip}}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^2}\hat{\phi}$;
   $\mathbf{B}_{\text{dip}}(\mathbf{r}) = \frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\theta}\right)$

Chapter 6 of Griffiths

1. Torque on magnetic dipole: $\mathbf{N} = \mathbf{m}\times\mathbf{B}$

2. Force on magnetic dipole: $\mathbf{F} = \nabla(\mathbf{m}.\mathbf{B})$

3. Bound currents due to magnetization: $\mathbf{J}_b = \nabla\times\mathbf{M}$ and $\mathbf{K}_b = \mathbf{M}\times\hat{\mathbf{n}}$

4. Definition of auxiliary field: $\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}$;

   Ampere's law for auxiliary field: $\int\mathbf{H}.d\mathbf{l} = I_{f\ enc}$; $\nabla\times\mathbf{H} = \mathbf{J}_f$

   Linear media: $\mathbf{M} = \chi_m\mathbf{H}$; $\mathbf{B} = \mu\mathbf{H}$, where $\mu = \mu_0(1+\chi_m)$

Chapter 7 of Griffiths

1. Ohm's law: $V = IR$; $\mathbf{J} = \sigma\mathbf{E}$

   Power in resistor: $P = I^2R = VI$

2. electromotive force (emf): $\mathcal{E} = \int(\mathbf{f}_s + \mathbf{E}).d\mathbf{l}$

3. Universal flux rule for motional emf and Faraday's law: $\mathcal{E} = -\frac{d\Phi}{dt}$, where $\Phi = \int\mathbf{B}.d\mathbf{a}$

4. Lenz's law: flux due to induced current opposes change in flux which induced the current itself

   Induced electric field: $\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$; $\int\mathbf{E}.d\mathbf{l} = -\frac{d\Phi}{dt}$

5. Mutual inductance: $\phi_2 = M_{21}I_1$; $M_{21} = M_{12} = \frac{\mu_0}{4\pi}\int\int\frac{d\mathbf{l}_1 d\mathbf{l}_2}{\mathscr{r}}$

6. Self-inductance: $\Phi = LI$; $\mathcal{E} = -L\frac{dI}{dt}$

7. Energy in magnetic fields: $W = \frac{1}{2}LI^2 = \frac{1}{2\mu_0}\int_{\text{all space}}B^2 d\tau$

8. Maxwell's "fix" of Ampere's law: add displacement current, $\mathbf{J}_d \equiv \epsilon_0\frac{\partial\mathbf{E}}{\partial t}$

9. Maxwell's equations: (i) $\nabla.\mathbf{E} = \frac{\rho}{\epsilon_0}$; (ii) $\nabla.\mathbf{B} = 0$; (iii) $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$; (iv) $\nabla \times \mathbf{B} - \mu_o \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J} \Rightarrow \nabla.\mathbf{J} = -\frac{\partial \rho}{\partial t}$

10. Maxwell's equations with magnetic charge: (i) $\nabla.\mathbf{E} = \frac{\rho}{\epsilon_0}$; (ii) $\nabla.\mathbf{B} = \mu_o \rho_m$; (iii) $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = -\mu_o \mathbf{J}_m$; (iv) $\nabla \times \mathbf{B} - \mu_o \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}_e \Rightarrow \nabla.\mathbf{J}_m = -\frac{\partial \rho_m}{\partial t}$ and $\nabla.\mathbf{J}_e = -\frac{\partial \rho_e}{\partial t}$

11. Reformulated Maxwell's equations in matter: (i) $\nabla.\mathbf{D} = \rho_f$; (ii) $\nabla.\mathbf{B} = 0$; (iii) $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$; (iv) $\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}_f$

12. Boundary conditions (linear media): (i) $\epsilon_1 E_1^{\perp} - \epsilon_2 E_2^{\perp} = \sigma_f$; (ii) $\mathbf{E}^{||} - \mathbf{E}^{||} = 0$; (iii) $B_1^{\perp} - B_2^{\perp} = 0$; (iv) $\frac{1}{\mu_1}\mathbf{B}_1 - \frac{1}{\mu_2}\mathbf{B}_2 = \mathbf{K}_f \times \hat{\mathbf{n}}$

Chapter 8 of Griffiths

1. Poynting's theorem: $\frac{dW}{dt} = -\frac{d}{dt}\int_{\mathcal{V}} \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_o}B^2\right) d\tau - \frac{1}{\mu_0}\int_{\mathcal{S}} (\mathbf{E} \times \mathbf{B})$; $\frac{\partial}{\partial t}(u_{\text{mech}} + u_{\text{em}}) = -\nabla.\mathbf{S}$

2. Poynting's vector (energy flux density, i.e., energy crossing per unit time, per unit area), $\mathbf{S} \equiv \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$

Chapter 9 of Griffiths

1. *One*-dimensional wave traveling along z-direction: wave equation $\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$, where v is speed of wave

   Sinusoidal wave of fixed angular frequency, ω, wave number k, wavelength λ: $f(z,t) = A\cos(kz - \omega t + \delta)$, with $v = \omega/k = \omega\lambda/(2\pi)$

2. EM waves in vacuum: $\nabla^2 \mathbf{E}$ (or $\mathbf{B}$) $= \frac{1}{c^2}\frac{\partial^2 \mathbf{E} \text{ (or } \mathbf{B})}{\partial t^2}$, where $c = 1/\sqrt{\mu_0 \epsilon_0}$

3. Plane wave traveling in direction of wave vector $\mathbf{k}$ (in vacuum) and polarized along direction $\hat{\mathbf{n}}$: $\mathbf{E}(\mathbf{r},t) = E_0 \cos(\mathbf{k}.\mathbf{r} - \omega t + \delta)\hat{\mathbf{n}}$ and $\mathbf{B}(\mathbf{r},t) = \frac{1}{c}E_0\cos(\mathbf{k}.\mathbf{r} - \omega t + \delta)\left(\hat{\mathbf{k}} \times \hat{\mathbf{n}}\right)$, with $\hat{\mathbf{n}}.\hat{\mathbf{k}} = 0$ and $\omega = kc$ ($\mathbf{E}$ and $\mathbf{B}$ are perpendicular to each other and transverse, i.e., perpendicular to the direction of propagation)

   Poynting vector along $\mathbf{k}$; intensity, $I = \frac{1}{2}c\epsilon_0 E_0^2$

4. EM waves in matter: $v = \frac{c}{n}$, where index of refraction $n = \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}}$, intensity, $I = \frac{1}{2}\epsilon v E_0^2$
   When waves pass from one medium to another, boundary conditions can be used to relate the transmitted and the reflected amplitudes to the incident one (and the three angles in case of oblique incidence)

5. General waveguide (along z-direction): $\mathbf{E}(x,y,z,t) = \mathbf{E}_0(x,y)\cos(kz - \omega t + \delta)$...
   no transverse electric and magnetic (TEM) wave

6. TE wave ($E_z = 0$) in a rectangular waveguide of sides a and b: $k = \sqrt{(\omega/c)^2 - \omega_{mn}^2}$, where $\omega_{mn} \equiv c\pi\sqrt{(m/a)^2 + (n/b)^2}$; wave velocity, $v = c/\sqrt{1 - (\omega_{mn}/\omega)^2}$ and group velocity, $v_g = c\sqrt{1 - (\omega_{mn}/\omega)^2}$

7. TEM wave in coaxial cable: $\mathbf{E}(s,\phi,z,t) = \frac{A\cos(kz-\omega t)}{s}\hat{s}$; $\mathbf{B}(s,\phi,z,t) = \frac{A\cos(kz-\omega t)}{cs}\hat{\phi}$ (i.e., perpendicular to each other), with $k = \omega/c$

Chapter 10 of Griffiths

1. Fields in terms of potentials: $\mathbf{B} = \nabla \times \mathbf{A}$; $\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$
2. Maxwell's equations for potentials: $\nabla^2 \mathbf{A} - \mu_0\epsilon_0 \frac{\partial^2 \mathbf{A}}{\partial t^2} - \nabla\left(\nabla.\mathbf{A} + \mu_0\epsilon_0\frac{\partial V}{\partial t}\right) = -\mu_0 \mathbf{J}$; $\nabla^2 V + \frac{\partial}{\partial t}\left(\nabla.\mathbf{A}\right) = -\frac{1}{\epsilon_0}\rho$
3. Use gauge freedom/transformation, i.e., $\mathbf{A}' = \mathbf{A} + \nabla\lambda$ and $V' = V - \frac{\partial \lambda}{\partial t}$, to go to Lorentz gauge, i.e., $\nabla.\mathbf{A} = -\mu_0\epsilon_0\frac{\partial V}{\partial t}$
4. Maxwell's equations for potentials in Lorentz gauge: $\nabla^2 \mathbf{A} - \mu_0\epsilon_0\frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{J}$; $\nabla^2 V - \mu_0\epsilon_0\frac{\partial^2 V}{\partial t^2} = -\frac{1}{\epsilon_0}\rho$
5. Solutions to above equations are retarded potentials: $V(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{r}',t_r)}{\boldsymbol{\mathscr{r}}}d\tau'$; $\mathbf{A}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{r}',t_r)}{\boldsymbol{\mathscr{r}}}d\tau'$, with $t_r \equiv t - \boldsymbol{\mathscr{r}}\ /c$

Chapter 11 of Griffiths

1. Oscillating (point) electric dipole, $\mathbf{p}(t) = p_0\cos(\omega t)\hat{\mathbf{z}}$, with $d \ll \lambda \ll r$
   $\mathbf{E} = -\frac{\mu_0 p_0 \omega^2}{4\pi}\left(\frac{\sin\theta}{r}\right)\cos\left[\omega\left(t - r/c\right)\right]\hat{\theta}$; $\mathbf{B} = -\frac{\mu_0 p_0\omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos\left[\omega\left(t-r/c\right)\right]\hat{\phi}$ (in phase, mutually perpendicular and transverse and ratio of amplitudes is c)
   Power radiated: $P = \frac{\mu_0 p_0^2\omega^4}{12\pi c}$
2. Power radiated by electric dipole moment of a general charge/current configuration: $P \approx \frac{\mu_0 \ddot{p}^2}{6\pi c}$