my notes

Compare plane wave with hollow guide (and coaxial)

— for all 3, start with ME in vacuum (rather no ρ, J free)

$$\Rightarrow \left(\partial^2/\partial x^2 + \partial^2/\partial y^2 + \frac{\partial^2}{\partial z^2}\right) \begin{matrix} E_x \\ E_y \\ \vdots \\ B_x \\ \vdots \end{matrix} = \underbrace{\frac{1}{c^2}}_{\text{rather } \mu\epsilon} \partial^2/\partial t^2 \, E_x$$

& "look" for monochromatic wave traveling in z-direction:

$\tilde{\mathbf{E}}(\vec{r},t) = \tilde{\mathbf{E}}_0(x,y)\, e^{i(kz-\omega t)}$ ... but $\tilde{E}_0$ so far has all 3 components

physical $\mathbf{E}$ is real component — no z, t here (but there's x, y dependence in general)

drop $\sim$ & 0

So far, don't have relation between $\{k, \omega\}$

— ① $\boxed{\text{Plane}}$ wave : $\tilde{E}$ constant (no x, y dependence)

plug into Maxwell's eqns. $\Rightarrow E_z = 0,\ B_z = 0$ (fully transverse)

and $(k = \omega/c)$: $\underbrace{\frac{\partial^2 E_{0\,x,y}}{\partial z^2}}_{k^2}$ (no z; no $\partial x^2, \partial y^2$) $= \underbrace{1/c^2\, \partial^2/\partial t^2}_{\omega^2} E_{0\,x,y}$ — no z, t dependence

(i.e., only 1 wave number for given ω)

e^{ikz} $e^{-i\omega t}$

— ② $\boxed{\text{Hollow}}$ wave guide : must have x, y dependence since confined ($E, B = 0$ outside) $\Rightarrow \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2 - \frac{\omega^2}{c^2}\right) \times E_{0\,x,y,z} = 0$

$\boxed{k \neq \omega/c}$ $\in$

and fully transverse (TEM) not allowed

[plug in Maxwell ... $(\vec{\nabla}\times\vec{E})_z = 0$, $(\vec{\nabla}\cdot\vec{E})_{xy} = 0$ ... $\Rightarrow \nabla^2_{2D} V = 0$ ...]

— w/ $\boxed{\text{rectangular}}$, $\partial/\partial x$, $\partial/\partial y$ acting on B_z, E_z gives $m, n \frac{\pi}{a, b}$ (in general expect quantization) $\Rightarrow$ $k = \omega/c \sqrt{1 - \omega^2/\omega^2_{mn}}$ cut-off ...

$\Rightarrow$ $\boxed{\text{many}}$ (discrete) k for given ω

③ Coaxial : fully transverse allowed ($\nabla^2 V = 0$, but 2 boundaries)

plug in Maxwell ($E_z = B_z = 0$) $\Rightarrow (k = \omega/c)$ & $\boxed{E \perp B}$ ... like plane, but there is x, y dependence