PHYS 410 Homework # 10

15.3. $v = 8\times10^3 m/s \Rightarrow \beta = \frac{v}{c} \doteq 2.67\times10^{-5}$

$$\Rightarrow \gamma = \frac{1}{\sqrt{1-\beta^2}} \doteq 1+\frac{1}{2}\beta^2 \doteq 1+3.56\times10^{-10}$$

$$t = \gamma(t'+\beta\frac{x'}{c}) \Rightarrow \Delta t = \gamma\Delta t'$$

$$\Rightarrow \Delta t' - \Delta t = (\frac{1}{\gamma}-1)\Delta t \doteq -3.56\times10^{-10}\times3.6\times10^3 s = -1.28\times10^{-6} s$$

$$\Rightarrow \frac{\Delta t'-\Delta t}{\Delta t}\times100\% = -3.56\times10^{-10}\times100\% = -3.56\times10^{-8}\%$$

15.6. Because the time interval in different frames depends only on the magnitude of the velocity, so we do not care whether he is out or back, the difference in time interval is:

$$\Delta t = \gamma\Delta t' \quad \text{(process is same with 15.3)}$$

$$\Rightarrow \gamma = \frac{\Delta t}{\Delta t'} = 3$$

$$\Rightarrow \beta = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{\frac{8}{9}} \doteq 0.94$$

$$\Rightarrow v \doteq 0.94c$$

15.11. $x' = \gamma(x-\beta ct) \Rightarrow \Delta x' = \gamma\Delta x$

$$\Rightarrow \gamma = \frac{\Delta x'}{\Delta x} = \frac{100}{80} = \frac{5}{4}$$

$$\Rightarrow \beta = \sqrt{1-\frac{1}{\gamma^2}} = \frac{3}{5}$$

$$\Rightarrow v = \frac{3}{5}c$$

15.17. $t_1' = \gamma(t-\beta x_1/c)$

$t_2' = \gamma(t-\beta x_2/c)$

(a) $\beta = \frac{V}{c} > 0$,

$$t_1'-t_2' = \gamma\beta(x_2-x_1)/c = \gamma\beta a/c > 0$$

(b) $\beta = \frac{V}{c} < 0$

$$t_1' - t_2' = \gamma\beta\,(x_2 - x_1)/c = \gamma\beta a/c < 0$$

So we could find that the time order of the two events is not definite, in fact, that's because the two events are in the "else where" or space-like zone relative to each other.

15.21

$$x = \gamma(x' + \beta c t')$$
$$t = \gamma\,(t' + \beta x'/c)$$
$$\Rightarrow \frac{dx}{dt} = \frac{\gamma(dx' + \beta c\,dt')}{\gamma\,(dt' + \beta\,dx'/c)}$$
$$= \frac{v' + \beta c}{1 + \beta \cdot v'/c}$$
$$= \frac{\frac{3}{4}c + \frac{c}{2}}{1 + \frac{1}{2} \times \frac{3}{4}} = \frac{10}{11}c \doteq 0.91c$$

15.49 $U = \gamma[\vec{v}, c]$

$$U \cdot U = \gamma^2(v^2 - c^2) = -c^2 \cdot \gamma^2 \cdot (1 - \frac{v^2}{c^2}) = -c^2$$

15.67. Suppose at initial time:

$$P_1 = [\vec{p}, E/c], \qquad P_2 = [-\vec{p}, E/c]$$

at final time:

$$P' = [\vec{P}', E'/c]$$

Conservation of 4-momentum gives us:

$$P' = [0, 2E/c]$$
$$\Rightarrow v' = 0, \quad Mc^2 = 2E$$

$$\Rightarrow M = \frac{2E}{c^2} = \frac{2\sqrt{p^2+m^2c^2}}{c^2} = 2\gamma m = \frac{10}{3}m \doteq 3.33m$$

15.79

$$\vec{F} = \frac{d\vec{p}}{dt}$$
$$= m\frac{d\gamma\vec{v}}{dt}$$
$$= \gamma m\frac{d\vec{v}}{dt} + \vec{v}\frac{d\gamma m}{dt}$$
$$= \gamma m\vec{a} + \frac{\vec{v}}{c^2}\frac{d\gamma mc^2}{dt} = \gamma m\vec{a} + \frac{\vec{v}}{c^2}\frac{dE}{dt}$$
$$= \gamma m\vec{a} + (\vec{F}\cdot\vec{v})\frac{\vec{v}}{c^2}$$

15.86 (a) If π^0 is at rest,

$P_{\pi^0} = [0, m_\pi c^2]$, $P_{\gamma_1} = [\vec{p}_{\gamma_1}, E_{\gamma_1}/c]$, $P_{\gamma_2} = [\vec{p}_{\gamma_2}, E_{\gamma_2}/c]$

$P_{\pi^0} = P_{\gamma_1} + P_{\gamma_2}$

$$\Rightarrow \begin{cases} \vec{p}_{\gamma_1} + \vec{p}_{\gamma_2} = 0 \\ E_{\gamma_1} + E_{\gamma_2} = m_\pi c^2 \end{cases}$$

$$\Rightarrow E_{\gamma_1} = E_{\gamma_2} = \frac{m_\pi c^2}{2} = 67.5 \text{mev}$$

(b) $|\vec{p}_\gamma| = \frac{E_\gamma}{c}$

Suppose the velocity of π^0 is $v = \beta c$, then we choose a new frame S', which moves with a velocity v. So in S', we reproduce the problem of (a), that is:

$$E_{\gamma_1}' = E_{\gamma_2}'$$

and from Lorentz transformation:

$$E_{\gamma_1}' = \gamma(E_1 - \vec{v}\cdot\vec{p}_1)$$
$$E_{\gamma_2}' = \gamma(E_2 - \vec{v}\cdot\vec{p}_2)$$
$$\Rightarrow \frac{E_{\gamma_1}'}{E_{\gamma_2}'} = \frac{E_1(1-\beta)}{E_2(1+\beta)} = 3\cdot\frac{1-\beta}{1+\beta} = 1$$
$$\Rightarrow \beta = \frac{1}{2}$$
$$\Rightarrow \vec{v} = \frac{c}{2}\hat{x}$$

15.107. First, note that $\mathcal{F}$ is antisymmetric, so we could only discuss the elements $\mathcal{F}_{\mu\nu}$ with $\nu \geqslant \mu$.

1° diagonal elements ($\mu = \nu$):

$$\mathcal{F}_{\mu\nu} = 0 = \Box_\mu A_\nu - \Box_\nu A_\mu = \Box_\mu (A_\mu - A_\mu)$$

2° $\nu = 4$:

$$F_{\mu\nu} = -\frac{E_\mu}{c} = (\nabla_\mu \phi + \frac{\partial A_\mu}{\partial t})/c = \Box_\mu A_4 - \Box_4 A_\mu = \Box_\mu A_\nu - \Box_\nu A_\mu$$

3° $\mu = 1$, $\nu = 2$:

$$F_{\mu\nu} = B_3 = (\nabla \times \vec{A})_3 = \frac{\partial}{\partial x_1} A_2 - \frac{\partial}{\partial x_2} A_1 = \Box_1 A_2 - \Box_2 A_1 = \Box_\mu A_\nu - \Box_\nu A_\mu$$

Similarly, we could prove this for $\mu = 1, \nu = 3$ and $\nu = 3, \mu = 2$.

To sum up, we prove $F_{\mu\nu} = \Box_\mu A_\nu - \Box_\nu A_\mu$.