Due Monday February 9

1) Consider the surface given by $z = a(x^2 + y^2)$. In this problem I want you to find an expression for the shortest distance between two points on this surface. Use as the coordinates x and y so that your path is parameterized by y(x).

   a) Show that the path length for a path from the point (x_1, y_1) to the point (x_2, y_2) is given by $S = \int_{x_1}^{x_2} dx\sqrt{1+(y')^2 + (2a)^2(x+yy')^2}$ where $y' \equiv \frac{dy}{dx}$ and $y_1 = y(x_1)$, $y_2 = y(x_2)$.

   b) Find the differential equation for y(x), which minimizes the path length.

2) Redo problem 1) in polar coordinates.

3) In class we used the principle of least time in optics to motivate the calculus of variations. However, we did not derive the path of a ray in an inhomogeneous medium. In this problem I want you to do this. Consider a medium whose velocity is a function of position, *v(x,y,z)* and find the differential equation for the path which minimizes the time to go from point (x_1, y_1, z_1) to (x_2, y_2, z_2). You may express the path as a function of *z; i.e. x(z)* and *y(z).*

4) A wire is bent in the form of a sine curve: $y = A\sin(kx)$. A frictionless bead of mass *m* slides on the wire. Gravity acts on the bead in the usual way.

   a) Show that the $\dot{y} = Ak\cos(k\,x)\,\dot{x}$

   b) Show that the Lagrangian is given by $L = \frac{m\left(1+(Ak\cos(kx))^2\right)\dot{x}^2}{2} - m\,g\,ASin(kx)$

   c) Find the equation of motion for this system