Final Exam Review

Accelerating Frames of Reference:

$$m\ddot{\vec{r}} = \vec{F} \underbrace{- m\vec{A}}_{\uparrow \text{ "pseudo-force"}} , \quad \vec{A} \text{ is acceleration of the frame of reference w/respect to an inertial system}$$

Rotating Frames, described by $\vec{\Omega}$ vector:

Two pseudoforces:

$$\vec{F}_{coriolis} \equiv 2m\dot{\vec{r}} \times \vec{\Omega}$$

$$\vec{F}_{centrifugal} \equiv m(\vec{\Omega} \times \vec{r}) \times \vec{\Omega}$$

Hamiltonian Mechanics

$$p_i \equiv \frac{\partial \mathcal{L}}{\partial \dot{q}_i}, \quad \text{Then} \quad \mathcal{H} = \sum_i p_i \dot{q}_i - \mathcal{L}, \quad i = 1, 2, 3, \ldots$$

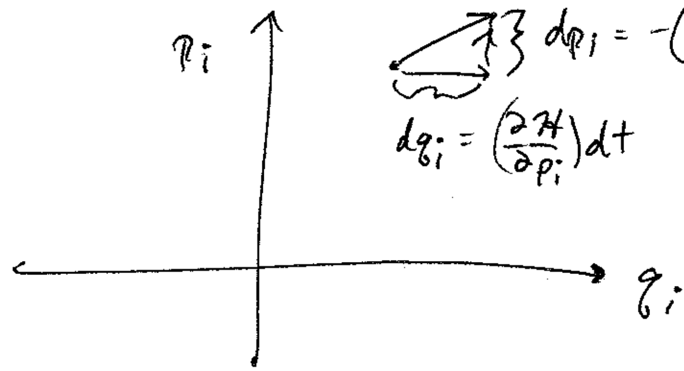
for each generalized coordinate.

Equations of Motion:

$$\boxed{\begin{aligned} \dot{q}_i &= \frac{\partial \mathcal{H}}{\partial p_i} \\ \dot{p}_i &= -\frac{\partial \mathcal{H}}{\partial q_i} \end{aligned}}$$

Conservation of Energy: The Hamiltonian is constant if the Lagrangian has no explicit time dependence. Also, the value of the Hamiltonian is exactly equal to the energy if the potential is velocity independent and the equations connecting the q_i and (x, y, z) do not depend on time.

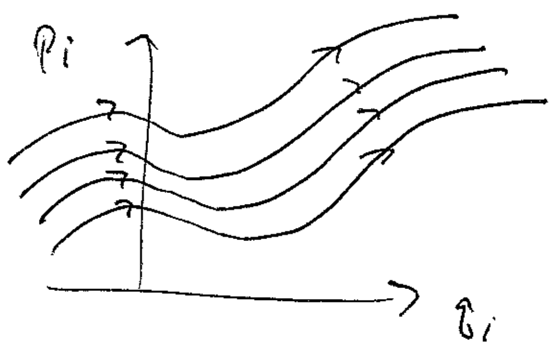
Phase Space:



$$dp_i = -\left(\frac{\partial \mathcal{H}}{\partial q_i}\right) dt$$

$$dq_i = \left(\frac{\partial \mathcal{H}}{\partial p_i}\right) dt$$

There can be no crossings in phase space.



Phase space density is conserved according to Liouville's Theorem

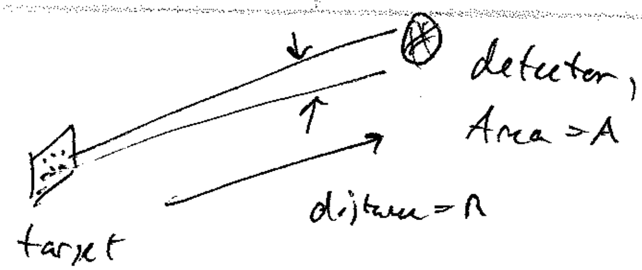
Rutherford Scattering

$$N_{scattered} = (N_{incoming})(n_{target})\, \sigma$$

n_{target}: # of target per unit volume

σ: cross-section, units = m^2 or cm^2 or barn ($10^{-24} cm^2$)

Solid Angle:



$$\Delta\Omega = \frac{A}{R^2}$$

Limit when $A \to \emptyset$, then $\Delta\Omega \to d\Omega = \dfrac{dA}{r^2}$

$$= \sin\theta \, d\theta \, d\phi$$

$$d\sigma(\text{scattering into } d\Omega) = \left(\frac{d\sigma}{d\Omega}\right) d\Omega$$

$\dfrac{d\sigma}{d\Omega}$ = differential cross section, $\sigma_{total} = \displaystyle\int \frac{d\sigma}{d\Omega} \, d\Omega$

$$= \int_{\emptyset}^{\pi} \sin\theta \, d\theta \int_{0}^{2\pi} d\phi \, \frac{d\sigma}{d\Omega}(\theta, \phi)$$

If b is the impact parameter which causes scattering angle θ, then

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta} \left| \frac{db}{d\theta} \right|$$

For Rutherford scattering (inverse square law),

$$b(\theta) = \frac{\gamma}{mv^2} \cot\left(\frac{\theta}{2}\right), \quad \gamma = kqQ$$

Then $$\frac{d\sigma}{d\Omega} = \left(\frac{kqQ}{4E\sin^2(\theta/2)} \right)^2$$

Special Relativity

time dilation: $\Delta t = \gamma \Delta t'$,

$$\boxed{\gamma = \frac{1}{\sqrt{1-\beta^2}}, \quad \beta = \frac{v}{c}, \quad \eta = \gamma\beta}$$

$\Delta t'$ = proper time,
time observed
between 2 events
in a frame where both events occur
at the same spatial location.

Length contraction: $\ell = \frac{\ell'}{\gamma}$, ℓ' = proper length
= length observed
in a frame where
the object is at rest.

Lorentz Transformation:

$$\begin{pmatrix} \Delta x' \\ c\Delta t' \end{pmatrix} = \begin{pmatrix} \gamma & -\eta \\ -\eta & \gamma \end{pmatrix} \begin{pmatrix} \Delta x \\ c\Delta t \end{pmatrix}, \quad \begin{matrix} \Delta y = \Delta y' \\ \Delta z = \Delta z' \end{matrix}$$

and

$$\begin{pmatrix} \Delta x \\ c\Delta t \end{pmatrix} = \begin{pmatrix} \gamma & \eta \\ \eta & \gamma \end{pmatrix} \begin{pmatrix} \Delta x' \\ c\Delta t' \end{pmatrix}$$

Space time interval: $(\Delta s)^2 \equiv (\Delta x)^2 - (c\Delta t)^2$
= invariant under
Lorentz Transformation.

$(\Delta s)^2 > \emptyset \Rightarrow$ "space-like interval"
$\Rightarrow$ 2 events are causally disconnected. Observers may disagree about the ordering of the events.

$(\Delta s)^2 = \emptyset \Longrightarrow$ "light-like interval"

$(\Delta s)^2 < \emptyset \Longrightarrow$ "time-like interval"
$\Rightarrow$ events can be causally related. All observers must agree on the ordering.

Formalism

We look for 4-vectors which transform according to the Lorentz Transformation: $A' = \Lambda A$,
$A =$ 4-vector.

An example is $(x, y, z, ct) \doteq X$
To calculate the "length" we do

$$(\Delta s)^2 = x^2 + y^2 + z^2 - (ct)^2 = X \cdot X$$

Another 4-vector is

$$p = (\gamma m\vec{v}, \gamma mc) = \text{energy momentum 4-vector}$$
$$= (\vec{p}, E/c)$$

Its "length" is called the invariant mass:

$$p \cdot p = \vec{p}^2 - (E/c)^2 = -m^2$$

This is usually written as

$$E^2 = (pc)^2 + (mc^2)^2 ,$$

Or, better yet, set $c = 1$. Then $E^2 = p^2 + m^2$

Also $\beta = \frac{pc}{E}$ or $\beta = \frac{p}{E}$

Also, but less useful, $\vec{p} = \gamma m \vec{v}$ and $E = \gamma mc^2$

Other 4-vectors:

4-velocity: $u \equiv \left(\gamma \frac{d\vec{x}}{dt}, \gamma c\right)$

4-wavenumber: $k \equiv \left(\vec{k}, \omega/c\right)$

Velocity transformation: $v_x' = \dfrac{v_x - V}{1 - v_x V/c^2}$

$$v_y' = \frac{v_y}{\gamma(1 - v_x V/c^2)}$$