Complete the following by hand (no assistance from computers!):

1. Use the Euler formula to expand $e^{i\theta}$ for real $\theta$.
2. Given the three Cartesian unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$, calculate the following:
   a. $\hat{x} \times \hat{y}$
   b. $|\hat{x}|$
   c. $\hat{x} \cdot \hat{y}$
3. Given the vectors $\vec{r} = (r_x, r_y, r_z)$ and $\vec{s} = (s_x, s_y, s_z)$, calculate the cross product vector $\vec{r} \times \vec{s}$ in terms of its Cartesian components.
4. Find the eigenvalues and eigenvectors of this matrix: $\tilde{A} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$.
5. What is the determinant of $\tilde{A}$ and how is it related to the eigenvalues?
6. What is the trace of $\tilde{A}$ and how is it related to the eigenvalues?
7. Given a scalar function of position $\chi(\vec{r})$ (e.g. the temperature distribution on the surface of the earth), what can we say is always true about the curl of the gradient of $\chi$?
8. Given a vector field $\vec{F} = k(x, 2y^2, 3z^3)$, where $k$ is a constant, calculate its curl, $\nabla \times \vec{F}$.
9. What is the general solution to the second-order linear differential equation $\ddot{x} = -\omega^2 x$, where $\omega$ is a real positive number?
10. What is the general solution to the second-order linear differential equation $\ddot{x} = +k^2 x$, where $k$ is a real positive number?
11. Given $ln(y) = b \ln(x)$, where $b$ is a constant, find $y$ as a function of $x$, $y(x)$.
12. Evaluate the integral $I = \int_2^3 5x \, dx$.
13. Expand $y(x) = \ln(1 + x)$ to second order for $x \ll 1$. Write the series expansion for $y(x) = \frac{1}{1-x}$ valid for $-1 < x < 1$.
14. Write down the differential volume element $d^3r$ in spherical coordinates. Use the figure below for definition of the spherical coordinates.