Measurements of the horizontal coefficient of restitution for a superball and a tennis ball

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When a ball is incident obliquely on a flat surface, the rebound spin, speed, and angle generally differ from the corresponding incident values. Measurements of all three quantities were made using a digital video camera to film the bounce of a tennis ball incident with zero spin at various angles on several different surfaces. The maximum spin rate of a spherical ball is determined by the condition that the ball commences to roll at the end of the impact. Under some conditions, the ball was found to spin faster than this limit. This result can be explained if the ball or the surface stores energy elastically due to deformation in a direction parallel to the surface. The latter effect was investigated by comparing the bounce of a tennis ball with that of a superball. Ideally, the coefficient of restitution (COR) of a superball is 1.0 in both the vertical and horizontal directions. The COR for the superball studied was found to be 0.76 in the horizontal direction, and the corresponding COR for a tennis ball was found to vary from −0.51 to +0.24 depending on the incident angle and the coefficient of sliding friction.
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I. INTRODUCTION

The physics of a bouncing ball has a long history and has been the subject of many articles in this journal.[1–4] Measurements are often reported on the coefficient of restitution (COR) of a ball for a vertical bounce, but very little information is available concerning the COR in the horizontal direction. The COR for a vertical bounce off a surface that remains at rest is defined as the ratio of the rebound speed to the incident speed. The horizontal COR can be defined for an oblique impact in terms of the horizontal components of the incident and rebound speeds of the contact point on the ball.

The physics of a ball incident at an oblique angle on a surface has been described theoretically by Garwin[1] and by Brody.[5] Garwin analyzed the bounce of a superball while Brody analyzed the bounce of a tennis ball. Despite the fact that both types of balls are relatively flexible and bounce well in the vertical direction, their bounce characteristics in the horizontal direction are dramatically different. Garwin assumed that the collision is perfectly elastic in both the vertical and horizontal directions, meaning that the vertical and horizontal components of the ball's speed at the contact point are both reversed by the bounce. In Brody's model, the collision is inelastic in the vertical direction and may be completely inelastic in the horizontal direction, in which case the contact point comes to rest and the ball then commences to roll during the impact. Both models are somewhat idealized, but they provide an adequate qualitative description of the bounce in each case. The results presented here appear to be the first measurements for any ball type that allows the two models to be evaluated quantitatively.

Interest in an oblique bounce is not just an intellectual exercise. The International Tennis Federation has recently approved new and expensive apparatus designed to measure the speed of any court surface, using a tennis ball projected onto the surface at high speed and at a low angle to the horizontal. Measurements of the rebound speed and angle provide sufficient information to determine the coefficient of sliding friction between the ball and the surface. A surface such as grass, which has a low coefficient of friction, is described as fast, while a surface such as clay, with a high coefficient of friction, is described as slow. The speed of a court affects not only the rebound speed of the ball but also the rebound spin and angle. Players recognize differences between these court surfaces by the way the ball tends to skid on grass or kick up at a steep angle on clay. Similarly, the rebound speed, spin, and angle of a ball struck by a tennis racket or a table tennis bat or a golf club or a cricket or baseball bat[6] is of interest in relation to the dynamics of these sports, both in terms of the flight aerodynamics and the subsequent bounce off the playing surface.

The spin of a ball in flight is difficult to measure with any technique other than high speed photography. High speed video cameras that capture images at up to 40,000 frames/s, at a cost of $40,000 or more are available. Most video cameras for consumer use capture images at 25 or 30 frames/s, but some now operate at 100 or 120 frames/s to allow for smooth slow motion playback. Digital video cameras can be used to transfer the images to a computer for further analysis. The digital camera used in the experiments described in this paper was a JVC 9600 which was purchased for $2400, including the image capture software.

II. QUALITATIVE FEATURES OF OBLIQUE BOUNCES

An obvious difference between a superball and a tennis ball is that the former bounces to a greater height when both are dropped from the same height. The bounce properties in the horizontal direction are less well known, but they are easily observed and easily explained. The differences are illustrated in Fig. 1 for a bounce on a hard surface such as a wood floor. A superball thrown at low speed onto the surface so that it is incident without spin at about 20° to the vertical will bounce forwards with a topspin at an angular speed typically around 5 rev/s. A superball incident at the same angle and speed but spinning backwards at around 10 rev/s will bounce backwards with topspin at around 10 rev/s. In that case, the spin, the horizontal velocity, and the vertical velocity all reverse direction as a result of the bounce.

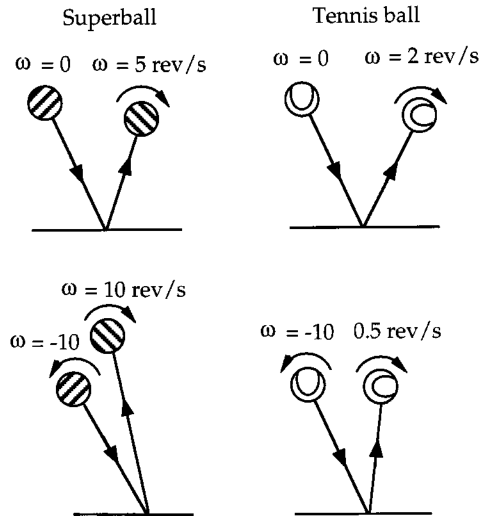

Fig. 1. Typical bounce parameters for a superball and a tennis ball incident at low speed on a hard surface.

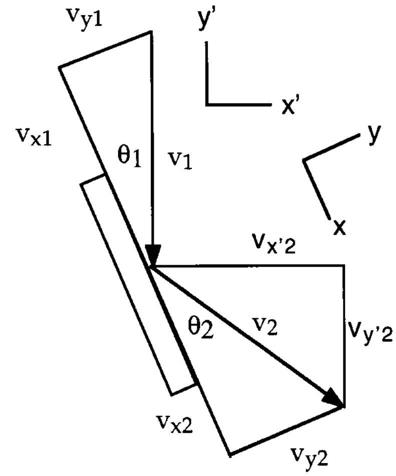

Fig. 2. The coordinate systems used to analyze the bounce of a ball incident vertically on a flat surface inclined at an angle to the vertical. The ball is incident at speed v_1 and angle θ_1 and rebounds at speed v_2 and angle θ_2.

A tennis ball incident on a hard surface without spin behaves in a similar manner to a superball, apart from the fact that it does not bounce as high and it spins at a slower rate. If it is incident with backspin and about 20° to the vertical, then the ball bounces almost vertically and with very little spin. These results can be described in terms of the net horizontal speed of the contact point, taking into account both the translational and rotational speed of the ball at that point. The horizontal speed of a superball at the contact point is approximately reversed by the bounce whereas the horizontal speed of a tennis ball at the contact point drops almost to zero—at least this is the case for a ball incident at typical throwing angles up to 60° to the vertical. A tennis ball that bounces on a tennis court is more commonly incident at angles less than 20° to the horizontal. In that case the ball usually slides throughout the impact and the speed of the contact point (or points) decreases with time but does not drop to zero.

It is impossible to observe by eye the behavior of a ball during a bounce because the bounce duration is only about 4 ms for both a superball and a tennis ball. A ball that is incident without spin and that rebounds at 10 rev/s rotates by about 0.02 revolutions or about 7° during the bounce. However, a flexible ball does not rotate as a rigid body during the bounce. The ball squashes in the vertical direction and it also deforms elastically in the horizontal direction if it is incident obliquely. Brody[5] avoided this problem by assuming that the impact speed was sufficiently low so that the ball remained approximately spherical with radius R, and that it was subject to a normal reaction force N and a horizontal friction force $F=\mu N$, where μ is the relevant coefficient of friction. A ball will usually commence to slide at the beginning of the impact period, in which case μ is the coefficient of sliding friction, μ_S. The friction force decelerates the ball in the horizontal (x) direction, reducing the v_x component of the center-of-mass speed of the ball. The friction force also exerts a torque about the center-of-mass, resulting in an increase in the angular speed ω in a clockwise direction. If ω increases to a point where $v_x=R\omega$, then any point on the ball in contact with the surface is momentarily at rest on the surface because it rotates backward at the same speed as it translates forward. A rigid ball will begin to roll under these conditions, in which case the friction force will suddenly decrease to a value $F=\mu_R N$, where μ_R is the coefficient of rolling friction. In general, a ball will slide throughout the entire duration of impact only if it is incident at an angle to the horizontal less than about 20° because the normal reaction force and hence the friction force is then usually too small to slow the ball to a point where it can roll.

A flexible ball has the potential to grip the surface rather than roll if the area of contact with the surface comes to rest. The bottom of the ball will remain at rest if the coefficient of static friction is sufficiently large. The dynamics of this interaction is quite complicated[7] and involves gripping and slipping in a similar manner to the grip-slip squeal of a tire as a car rounds a bend. The details can be ignored if one assumes that the value of e_x, the COR in the horizontal direction, is determined experimentally. For the same reason, the COR in the vertical direction is rarely calculated theoretically because it is much easier to measure it experimentally. If one obtains e_x in this way, then Garwin's model can be modified (as described below) to describe interactions of a rigid or a flexible ball with a flexible surface. Examples include the bounce of a table tennis ball on a bat with a rubber or foam surface, and the bounce of a tennis ball on the strings of a racquet.[8]

III. GARWIN'S MODEL

Consider a ball of mass m and radius R incident at speed v_1, angular velocity ω_1, and at an angle θ_1 on a flat surface, as shown in Fig. 2. The surface may be horizontal, or it may be inclined at an angle to the horizontal, as in the experiments described below. For simplicity, it is assumed that the mass of the surface is infinite and that the impact force is much larger than the gravitational force. The dynamics of the collision are described by the relations $N=m\,dv_y/dt$ and $F=-m\,dv_x/dt$, where N is the normal reaction force, F is the force acting parallel to the surface, and v_x, v_y are the velocity components of the center-of-mass of the ball parallel and perpendicular to the surface, respectively. In Garwin's model[1] the equations of motion are not needed explicitly, because the collision can be described in terms of the vertical

(e_y) and horizontal (e_x) values of the COR, together with conservation of angular momentum about the contact point. Garwin assumed that $e_y = e_x = 1$, but his model is easily extended to cases where e_y and e_x are less than 1. Referring to Fig. 2, we define

$$e_y = -\frac{v_{y2}}{v_{y1}}, \tag{1}$$

where the subscripts 1 and 2 denote conditions before and after the collision, respectively, and where e_y is between 0 and 1 (v_{y1} being negative). Similarly, e_x can be defined by the relation

$$e_x = -\frac{v_{x2} - R\omega_2}{v_{x1} - R\omega_1}, \tag{2}$$

where $v_x - R\omega$ is the net horizontal speed of a point at the bottom of the ball. Unlike e_y, e_x can be positive or negative. If a ball is incident at sufficiently small θ_1 and without spin, then it can slide throughout the impact without rolling and will bounce with $R\omega_2 < v_{x2}$, in which case $e_x < 0$. A value $e_x = -1$ corresponds to a bounce on a frictionless surface, where $v_{x2} = v_{x1}$ and $\omega_2 = \omega_1$.

The horizontal friction force F exerts a torque $FR = I\,d\omega/dt$, where I is the moment of inertia about an axis through the center of the ball, so that

$$I\frac{d\omega}{dt} + mR\frac{dv_x}{dt} = 0. \tag{3}$$

It is assumed that N acts along a line through the center-of-mass and does not exert a torque on the ball. Conservation of angular momentum about a point at the bottom of the ball is therefore described by the relation

$$I\omega_1 + mRv_{x1} = I\omega_2 + mRv_{x2}, \tag{4}$$

provided that the ball is spherical before and after the collision. The moment of inertia of a spherical ball is given by $I = \alpha mR^2$, where $\alpha = 2/5$ for a uniform solid sphere and $\alpha = 2/3$ for a thin spherical shell. Equations (1)–(4) can be solved to show that

$$v_{x2} = \frac{(1 - \alpha e_x)v_{x1} + \alpha(1 + e_x)R\omega_1}{(1 + \alpha)}, \tag{5}$$

$$v_{y2} = -e_y v_{y1}, \tag{6}$$

and

$$\omega_2 = \frac{(1 + e_x)v_{x1} + (\alpha - e_x)R\omega_1}{R(1 + \alpha)}. \tag{7}$$

If $\omega_1 = 0$ and $e_x = 1$, then $v_{x2} = 0.2v_{x1}$ for a hollow shell and $v_{x2} = 0.429v_{x1}$ for a solid sphere. The corresponding spin values are $R\omega_2/v_{x2} = 6$ for a shell and $R\omega_2/v_{x2} = 10/3$ for a solid sphere. In both cases the ball spins much faster than one would expect from the rolling condition $R\omega_2 = v_{x2}$. At the end of the bounce, a ball with $e_x = 1$ will therefore slide backward on the surface, due to the recovery of elastic energy stored in the horizontal direction. Alternatively, if $\omega_1 = 0$ and $e_x = 0$, then $v_{x2} = 0.6v_{x1}$ for a shell and $v_{x2} = 0.714v_{x1}$ for a solid ball. In both cases, $R\omega_2 = v_{x2}$, meaning that the ball rolls at the end of the duration of the impact and there is no energy recovery or no energy stored elastically in the horizontal direction.

Because e_x is close to 1 for a superball and close to 0 for a tennis ball, a superball will bounce with a smaller v_{x2} component than a tennis ball when $\omega_1 = 0$ and v_{x1} is the same for both balls. Because v_{y2} is larger for a superball (for the same v_{y1}), a superball will bounce at a steeper angle (closer to the vertical) than a tennis ball. It is also easy to show that a superball with the same radius as a tennis ball and the same value of v_{x1} will bounce with greater spin, by a factor of 2.38 if $\omega_1 = 0$. If the superball is smaller, it will spin even faster.

IV. BRODY'S MODEL

Of the two models, the one that best describes the tennis ball results presented below is Brody's model. Consequently, this model is now considered in more detail, with a slight modification to allow for finite wall thickness of the ball. During the impact, the radius of the ball varies as a result of its compression. As a first approximation, it will be assumed that the compression is small and the radius, R, is essentially the same as that for a spherical ball. The angular acceleration of the ball is given by $FR = I\,d\omega/dt$, where I is the moment of inertia about an axis through the center-of-mass of the ball. A tennis ball can be approximated as a thin spherical shell with $I = 2mR_1^2/3$, where R_1 is the average radius of the shell. The wall is typically about 6 mm thick, including a 3-mm-thick outer cloth cover. For the calculations presented below, we will take $R = 33.0$ mm and $R_1 = 30$ mm. The effect of the finite wall thickness is that I is reduced by about 17% compared with a ball of negligible wall thickness. The rebound spin is also increased by about 17%, provided that the ball slides throughout the impact. If the ball starts rolling before it rebounds, then a reduction in I will cause the ball to start rolling earlier during the impact, but it has only a relatively small effect on the final rebound spin.

The rebound speed v_2, spin ω_2, and angle θ_2 can be determined by taking the time integrals of N and F over the impact period, τ, so that

$$\int_0^\tau F\,dt = -m(v_{x2} - v_{x1}), \tag{8}$$

$$\int_0^\tau N\,dt = m(v_{y2} - v_{y1}), \tag{9}$$

and

$$R\int_0^\tau F\,dt = I(\omega_2 - \omega_1), \tag{10}$$

where $v_{y1} < 0$ because the ball is incident in the negative y direction. If the ball slides throughout the impact, then $F = \mu_S N$, in which case it can be shown from Eqs. (8)–(10) that

$$v_{x2} = v_{x1} + \mu_S(1 + e_y)v_{y1}, \tag{11}$$

$$v_{y2} = -e_y v_{y1}, \tag{12}$$

and

$$\omega_2 = \omega_1 - 1.5\mu_S R(1 + e_y)v_{y1}/R_1^2. \tag{13}$$

The ball will begin rolling just at the end of the impact period if $v_{x2} = R\omega_2$, in which case

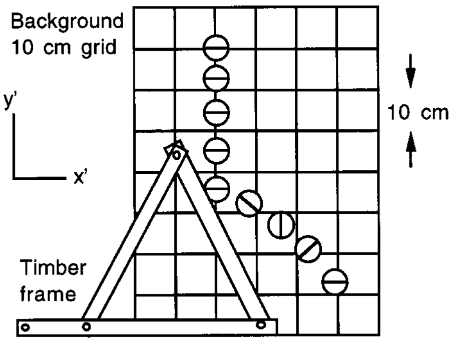

Fig. 3. Experimental arrangement showing the position and orientation of a ball at equal time intervals when incident vertically on an inclined surface.

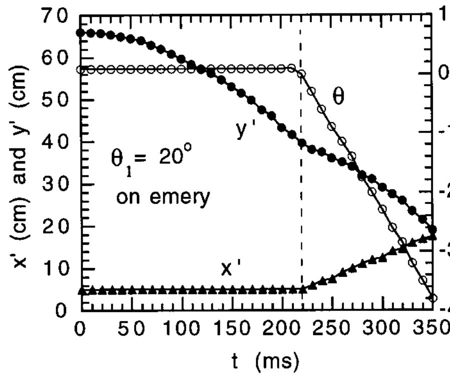

Fig. 4. Data obtained for a tennis ball dropped from a height of 25 cm above a timber platform inclined at 20° to the vertical. The surface was covered with a fine grain emery paper to increase the coefficient of sliding friction. The quantity x' is the horizontal coordinate of the center of the ball in the laboratory (that is, reference grid) frame; y' is the vertical coordinate and θ is the rotation angle. Each data point is separated by 10 ms.

$$v_{x2}=\frac{R\omega_1+(1.5R^2/R_1^2)v_{x1}}{(1+1.5R^2/R_1^2)}. \tag{14}$$

Rolling commences at the end of the impact period if

$$\mu_S=\frac{R\omega_1-v_{x1}}{(1+e_y)(1+1.5R^2/R_1^2)v_{y1}}. \tag{15}$$

If μ_S is larger than the right-hand side of Eq. (15), then the ball will start to roll before the end of the impact period. If $\mu_R=0$, there is no additional change in v_x or ω while the ball rolls, and hence v_{x2} and ω_2 are the same as if the ball started rolling at the end of the impact period. A small change in v_x and ω results if μ_R is finite, but the change can be neglected if $\mu_R<0.05$ as in the experiments described below.

If $R_1=R$ and $\omega_1=0$, and if the ball enters a rolling mode, then $v_{x2}=0.6v_{x1}$, which is the result obtained above and by Brody.[5] If $\omega_1=0$, $R=33.0$ mm, and $R_1=30$ mm, then $v_{x2}=0.645v_{x1}$ if the ball enters a rolling mode. A tennis ball incident on a horizontal surface will therefore slow down in the horizontal direction by at most 35%, provided the incident spin is zero. However, if the ball is incident with backspin, then the reduction in horizontal speed can be much larger. According to Eq. (14), the ball can even bounce backwards if ω_1 is sufficiently large and negative.

V. EXPERIMENTAL ARRANGEMENT

The arrangement used in this experiment is shown in Fig. 3. A slightly used 57-g tennis ball was dropped vertically through a measured height of either about 30 or about 60 cm to impact on a 3.8-kg timber platform which was mounted at various angles to the horizontal by means of a supporting frame resting on a solid floor. The ball bounced either directly on the polished timber platform, or on emery paper taped firmly onto the upper surface, or onto a 32 cm×25 cm sample of Rebound Ace clamped to the platform. Rebound Ace is the tennis court surface used at the Australian Open, and consists of a 1-mm-thick, slightly rough acrylic upper layer with a 6-mm-thick rubber backing. For each of these surfaces, the platform was inclined so that the ball could impact the surface at an angle $\theta_1=20°$, 45°, or 90°.

A straight line was drawn across a ball diameter with a felt pen so that the rotation angle of the ball could be recorded, as shown in Fig. 3. The ball was dropped vertically with zero initial speed, zero initial spin, and with the line on the ball facing the camera and passing through the visible center of the ball. The impact was recorded at 100 frames/s, and with an exposure time of 1/500 s, using two 100-W spotlights to provide extra illumination and to minimize shadows. Because the impact duration was about 4 ms, a detailed study of ball behavior during an impact was not possible. A 10-cm grid was drawn on a large sheet behind the ball in order to calibrate the horizontal and vertical distance scales on the video image. The grid was located 3-cm behind the ball and the camera was positioned 2 m in front of the ball to minimize parallax errors.

For a given surface, angle of incidence, and drop height, several bounces were recorded, and at least ten images before and after each bounce were transferred to a personal computer. The images were then printed one frame at a time and manually digitized, as shown by the example in Fig. 4. The latter process was very slow. It is now possible to download images in real time at 50 frames/s using almost any digital video camera and to analyze each frame using either free or commercially available software,[9] but this was not possible with the camera available. Because the background grid was aligned parallel and perpendicular to the floor, the velocity components of the ball immediately before and immediately after the impact were first determined in terms of the grid coordinate system, (x', y'). In this system, the horizontal component of the ball speed is unaffected by the gravitational force, and could be determined to within 2% using a linear fit to the data. The incident vertical component of the ball speed was found to agree, to within 2%, with the expected result $v_1=(2gh)^{1/2}$, where h is the drop height. The rebound spin of the ball is also unaffected by the gravitational force and could be determined to within 1%. The vertical component of the rebound velocity was more difficult to determine accurately because it changes with time as a result of the gravitational acceleration, and small errors in locating the center of the ball immediately after the impact generate relatively large errors in determining the initial rebound speed. Consequently, at least ten data points following the impact were analyzed by fitting a quadratic to account for the gravitational acceleration. Using this technique, the vertical component of the rebound velocity immediately following the impact was determined to within 3%. The velocity data

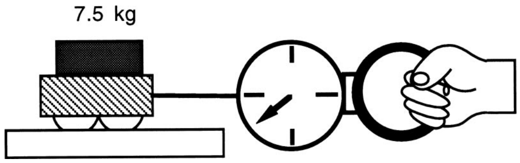

Fig. 5. Arrangement used to measure the coefficient of sliding friction.

in the grid coordinate system was then used to calculate the velocity components parallel and perpendicular to the inclined surface.

VI. MEASUREMENTS OF THE FRICTION COEFFICIENTS

The coefficient of sliding friction between a tennis ball and each of the three different surfaces was measured using the apparatus shown in Fig. 5. A square wood box was constructed so that four tennis balls could be squashed into the box and dragged across each surface without the balls rolling. The box and balls weighed 0.67 kg, and additional lead masses were placed on top of the box to give a total mass of 7.5 kg. The force required to drag the balls across the surface, at a constant low speed, was measured with a spring balance. These measurements gave $\mu_S = 0.23 \pm 0.02$ for the polished wood surface, $\mu_S = 0.60 \pm 0.03$ for Rebound Ace, and $\mu_S = 0.67 \pm 0.03$ for the emery paper.

The coefficient of rolling friction was measured by placing four tennis balls on the surface, and then placing a polished wood platform on top of the balls. Additional lead weights were placed on top of the platform to give a total load of 15 kg. The platform was then pulled horizontally at a constant low speed so that the balls could roll between the platform and the surface. The horizontal force was measured with a spring balance. These measurements gave $\mu_R = 0.04 \pm 0.005$ for all three surfaces.

The coefficient of restitution was also measured, by dropping the ball vertically onto the timber platform resting horizontally on the floor, and recording the rebound height using the video camera. For drop heights from 0.3 to 1.0 m, $e = 0.77 \pm 0.01$ for all three surfaces.

VII. TENNIS BALL RESULTS

A summary of results extracted from the data shown in Fig. 4 is given in Table I, which includes (a) the velocity components parallel and perpendicular to the surface, (b) the time integrals of F and N as given by Eqs. (8) and (9), and (c) a time-average coefficient of friction, defined by the relation $\mu_A = \int F\,dt / \int N\,dt$. Provided the ball slides throughout the impact, so that $F = \mu_S N$, then μ_A provides a measure of the time-average value of μ_S under actual impact conditions. However, if the ball starts to roll during the impact, then μ_A is lower than μ_S because $F = \mu_R N$ during the rolling stage. The experimental errors in all the derived quantities shown in Table 1 were determined from the ''worst'' combinations of the measured quantities $v_{1y'}$, $v_{2x'}$, $v_{2y'}$, and ω_2, and are therefore overly pessimistic. From these results, it can be seen that

(a) the coefficient of restitution, $e_y = v_{2y}/v_{1y} = 0.79 \pm 0.05$, is consistent with the value 0.77 ± 0.01 measured at perpendicular incidence;
(b) μ_A is consistent with the value of μ_S (0.67 ± 0.02) measured under quasistatic conditions;
(c) the ball spins 41% faster than one would expect if it was rolling at $\omega_2 = v_{2x}/R = 37.3$ rad at the end of the impact period; and
(d) $e_x = 0.24 \pm 0.03$ for this bounce.

Results for $h = 60$ cm are shown in Table II to provide comparisons between different surfaces and different angles of incidence. The results in Table II are given for a typical individual bounce on each surface at each angle of incidence, and do not represent averages taken over several different bounces. From this data we can see the following:

(a) μ_A is much less than μ_S for impacts at 45° on the emery and Rebound Ace surfaces, indicating that the ball commenced rolling well before it bounced off the surface.

(b) v_2 is always less than v_1, as expected.

(c) θ_2 is generally larger than θ_1, except for the low angle bounce on wood.

(d) ω_2 increases as θ_1 increases on the low μ_S wood surface, but it decreases as θ_1 increases on the high μ_S emery and Rebound Ace surfaces.

(e) At $\theta_1 = 45°$, ω_2 is larger on the low friction wood surface than on the high μ_S surfaces. The latter result is partly due to the slightly higher incident speed on the wood surface and partly the result of rolling friction acting for a longer period on the high μ_S surfaces.

(f) At $\theta_1 = 45°$, $R\omega_2 \approx v_{x2}$, as one would expect if the ball commenced rolling during the bounce. However, $R\omega_2 > v_{x2}$ on the wood surface, indicating that a small amount of energy was stored in a direction parallel to the surface.

(g) At $\theta_1 = 20°$, $R\omega_2 < v_{x2}$ on wood and $R\omega_2 > v_{x2}$ on the emery and Rebound Ace surfaces. The result on wood indicates simply that the ball did not commence rolling during the bounce. The results on emery and Rebound Ace indicate that friction was sufficiently large to allow for some storage and recovery of elastic energy in a direction parallel to the surface.

Results obtained at lower incident speeds, as well as the

Table I. Results obtained from Fig. 4 ($\theta_1 = 20°$ on emery) with v in m/s. The units of impulse, $\int F\,dt$ are Newton seconds (N s).

$v_{x'1}$	$v_{y'1}$	$v_{x'2}$	$v_{y'2}$	θ_1	θ_2
0	2.24 ± 0.04	0.99 ± 0.02	0.95 ± 0.03	$20° \pm 0.5°$	$26.2° \pm 1.5°$
v_{x1}	v_{y1}	v_{x2}	v_{y2}	v_1	v_2
2.10 ± 0.04	0.77 ± 0.014	1.23 ± 0.03	0.61 ± 0.03	2.24 ± 0.04	1.37 ± 0.03
ω_2	v_{y2}/v_{y1}	$\int F\,dt$	$\int N\,dt$	μ_A	$R\omega_2/v_{x2}$
52.4 ± 0.5 rad/s	0.79 ± 0.05	0.050 ± 0.004	0.079 ± 0.002	0.64 ± 0.05	1.41 ± 0.05

Table II. Results for three different surfaces (v in m/s, ω_2 in rad/s).

Surface	θ_1	v_1	v_2	θ_2	ω_2	$R\omega_2$	v_{x2}	v_{x2}/v_{x1}	e_y	e_x	μ_A	μ_S
Wood	20°	3.72	3.09	19.2°	34.9	1.15	2.92	0.84	0.80	−0.51	0.25	0.23
Wood	45°	3.80	2.53	50.4°	58.2	1.92	1.61	0.60	0.73	0.11	0.23	0.23
Emery	20°	3.53	2.24	27.2°	78.5	2.59	1.99	0.60	0.84	0.18	0.59	0.67
Emery	45°	3.70	2.66	51.3°	49.9	1.65	1.66	0.63	0.79	−0.01	0.20	0.67
R. Ace	20°	3.35	2.13	23.4°	76.7	2.53	1.96	0.62	0.74	0.18	0.60	0.60
R. Ace	45°	3.60	2.66	49.3°	50.6	1.67	1.74	0.68	0.79	−0.03	0.18	0.60

results shown in Table II, are plotted in Figs. 6 and 7 to compare with theoretical calculations of Sec. IV (Brody's model). The theoretical curves were evaluated with $R = 33.0$ mm, $R_1 = 30$ mm, $m = 57$ g, and $e_y = 0.77$. In theory, the ratios v_2/v_1, θ_2/θ_1, and $R\omega_2/v_1$, shown in Figs. 6 and 7, are all independent of v_1 if e_y is independent of v_1. The experimental data points in Figs. 6 and 7 were plotted assuming that μ_S was equal to μ_A for the wood surface. The value of μ_A was slightly different for each bounce, but all values were consistent with the value $\mu_S = 0.23 \pm 0.02$ as described in Sec. V. For the emery and Rebound Ace surfaces, where μ_S cannot be determined reliably from the bounce data, μ_S was assigned a value determined by loading the ball with a 7.5 kg mass (that is, $\mu_S = 0.60 \pm 0.03$ for Rebound Ace and $\mu_S = 0.67 \pm 0.03$ for emery). In order to separate the data points more clearly in Figs. 6 and 7, the latter points were plotted with a small spread in the assigned values of μ_S. For example, the data points for emery are plotted with $\mu_S = 0.64$ for the higher incident speed bounces, and $\mu_S = 0.67$ for the lower speed bounces. Similarly, the data points for Rebound Ace are plotted with $\mu_S = 0.59$ or 0.61.

The experimental results in Figs. 6 and 7 agree remarkably well with Brody's model, given the simplifying assumptions. The only significant departure is that the ball spin and rebound angle are higher than predicted for the bounces in Fig. 6 on high μ_S surfaces at a low angle of incidence. Both of these effects indicate that some of the impact energy is stored elastically in horizontal deformation of the ball and that it is partially recovered during the rebound, giving $e_x \approx 0.2$. Recovery of elastic energy in the horizontal direction has no effect on v_{y2}, but v_{x2} is reduced and hence θ_2 is increased as described in Sec. III.

An alternative explanation for the higher than predicted spin is that the normal reaction force might act vertically through a point behind instead of through the center-of-mass. This would provide an additional clockwise torque on the ball. Such an effect would be expected if the bottom of the ball grips the surface while the top of the ball continues to move horizontally at a slightly higher speed. If so, then the ball would tend to lean forwards during the bounce. In fact, it is precisely this effect that leads to elastic deformation or shear in the horizontal direction. However, if one incorporates this additional torque into Brody's model, then v_{x2} is increased and hence θ_2 is decreased. A decrease in θ_2 is in the wrong direction to explain the rebound angle results in Fig. 6. If the effect does occur, then the recovery of elastic energy appears to be a more dominant effect.

An additional complication is that any dynamic distortion of a ball from its initial spherical shape leads to high frequency oscillations of the ball. For a tennis ball,[4] the fundamental vibration frequency is about 500 Hz. Consequently, a ball impacting a surface at an oblique angle vibrates both horizontally and vertically for about two full cycles of oscillation during the impact. High speed video film taken by the International Tennis Federation shows that the oscillations are large, typically about 10 mm in amplitude for a high speed impact on concrete or on the strings of a racquet. Similar high frequency oscillations can be observed on the surface of a superball if one attaches a piezo disk to the surface.

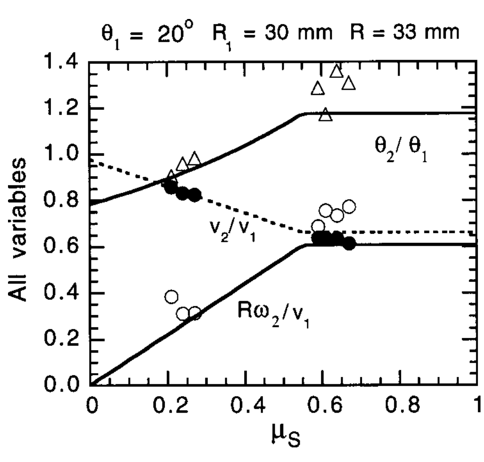

Fig. 6. Theoretical and experimental results for a tennis ball incident at 20° on three different surfaces. (△=experimental values of θ_2/θ_1. ●=v_2/v_1. ○=$R\omega_2/v_1$.)

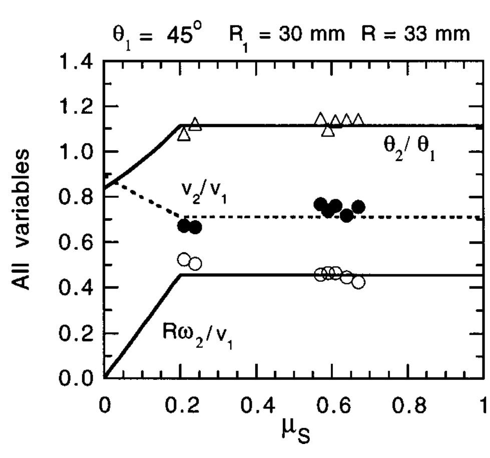

Fig. 7. Theoretical and experimental results for a tennis ball incident at 45° on three different surfaces. (△=experimental values of θ_2/θ_1. ●=v_2/v_1. ○=$R\omega_2/v_1$.)

VIII. SUPERBALL RESULTS

Measurements were taken of the bounce of a large superball of mass 102.5 g and diameter 60.0 mm, by filming the bounce when it was thrown at low speed onto a smooth, hard wood floor. A white line drawn across a diameter was used to measure the rotation speed. The ball was thrown using both hands to impart backspin, so that it was incident at about 20° to the vertical. Thirty bounces were filmed and three almost identical bounces were analyzed. The parameters for one particular bounce were $v_{y1} = -4.13$ m/s, $v_{x1} = 1.18$ m/s, $\omega_1 = -67.0$ rad/s, $v_{y2} = 3.57$ m/s, $v_{x2} = -0.625$ m/s, and $\omega_2 = 60.0$ rad/s. The ball was therefore incident at 15.9° to the vertical and it bounced at 9.9° to the vertical, with $e_y = 0.86$. For this bounce, $R\omega_2 = 2.88 v_{x2}$ and $e_x = 0.76$. Essentially the same result was obtained for the other two bounces.

The coefficients e_y and e_x are both less than the ideal value 1.0, but both are significantly larger than the corresponding values for a tennis ball. It is possible that e_x will vary with incident angle and bounce surface, but this possibility was not investigated for the superball. The coefficient of static friction for the superball on the wood floor was measured to be 0.52 ± 0.04 using four identical superballs squashed in a box as described in Sec. VI. The corresponding coefficient with four tennis balls in the box was 0.15 ± 0.02. This result is consistent with the fact that a superball tends to stick to the surface during a bounce whereas a tennis ball is more likely to roll.

IX. DISCUSSION

One of the difficulties underlying any quantitative analysis of ball sports is obtaining accurate data on the bounce properties of both the ball and the surface on which it bounces. In the absence of such data, one can adopt a theoretical bounce model such as the Brody or the Garwin model, but neither model on its own is capable of explaining some of the features that are observed in practice. Departures from bounce model predictions are likely to be observed in all ball sports, and can be illustrated with an example from tennis with which the author is more familiar. If one calculates the rebound speed, spin, and angle of a tennis ball incident on a tennis court and then combines that information with measured lift and drag coefficients to determine the resulting ball trajectory, then the result is not always consistent with common observations. For example, players and commentators universally agree that a ball bounces much more slowly and much higher off a clay court than off grass, despite the fact that calculations based on available data show that the differences should be relatively small. A ball served at 200 kph (124 mph) lands on the court at a speed of about 150 kph, at an angle of incidence of about 12° with $v_{y1} \approx 8$ m/s. The ball will take about 0.59 s to reach the opponent's baseline if it is served on a grass court with $\mu_S = 0.6$. On a clay court with $\mu_S = 0.8$ the ball will take 0.02 s longer according to the Brody bounce model. This time difference appears to be too short to make a significance difference to the nature of the game, given that a slightly slower serve speed on grass would have the same effect, yet players see the two surfaces as being quite different.

There are circumstances where a difference of 1% in ball speed or spin can make a big difference to a good player, but to an experimental physicist, a 1% difference is usually not significant. In tennis, a difference of 1% in ball speed can translate to a difference in ball position of several feet, so a 1% difference can win or lose a match. However, this difference does not seem to be the dominant factor determining the difference between clay and grass surfaces. Players compensate for the slower clay surface not by serving the ball faster but by serving slower. The average first serve speed for men playing on the grass courts at Wimbledon is about 185 kph. At the French Open, which is played on clay, the average first serve speed is only 160 kph. The reduction in serve speed in itself could be seen to account for the observation that the ball bounces much more slowly on clay, but players on clay serve with more topspin, which necessarily reduces the serve speed. In order to impart topspin to a ball, the ball must be incident obliquely on the strings (in the racquet frame of reference) rather than at normal incidence, with the result that the ball acquires rotational energy at the expense of translational energy.

There are several reasons why players might serve more slowly on a clay court, or faster on a grass court, but there are insufficient data to explain the difference in serve speeds with certainty. One possibility is that the maximum horizontal coefficient of restitution on clay is not zero, as assumed in the Brody model, but it is larger than zero as observed above. If e_x was sufficiently large, then v_{x2} could be significantly smaller than predicted by the Brody model in which case clay courts could indeed be slower than expected. A ball served at around 200 kph is incident at a relatively low angle on the court, typically about 12°. Such a ball should slide throughout the impact and at no time will the bottom of the ball come to rest or grip the surface as in Garwin's model, regardless of whether the ball is served on grass or clay. Consequently, e_x should be negative in this case and the ball will slow down by about the same amount on both surfaces, as predicted by the Brody model. However, a ball served at 160 kph with heavy topspin is incident on the court at an angle of about 16°. If the coefficient of sliding friction is larger than about 0.7, the ball might enter a rolling mode ($e_x = 0$) or it might grip the surface ($e_x > 0$). If the ball grips the surface, it will kick up at a steep angle and reach the peak of its trajectory slightly behind the baseline. If the ball slides throughout the bounce, it will rebound at a smaller angle and reach the peak of its trajectory near the back fence.

An alternative explanation for the slow serve speeds at the French Open is that e_y on clay may be larger than the value 0.75 specified by the rules of tennis for a 100-in. (2.54-m) drop on a hard surface. The values of e_y given in Table II were obtained at low ball speeds and are not directly relevant to the problem because it is known that e_y decreases as the ball speed increases, at least for a vertical bounce. However, observations of players in action indicate that e_y could be as large as 0.85 if a ball is incident obliquely and with topspin. A player serving a ball with reduced speed but with heavy topspin on a clay court can get the ball to bounce to around head height. This strategy is favored by players because a ball arriving at head or shoulder height is more difficult to return than one arriving at waist height. A bounce around head height is considerably higher than one would expect if $e_y = 0.75$. A ball served at around 160 kph with heavy topspin is incident on the court at a vertical speed of about 8 m/s. If it bounces with $e_y = 0.75$, then a simple estimate of the bounce height ignoring aerodynamics indicates that the ball should bounce to a height of about 1.8 m (about head height). However, if one includes the effect of the Magnus force arising from ball spin, then the ball should bounce slightly above waist height. It seems that the incident spin of a ball may enhance e_y if the ball is spinning fast enough. One might expect that if a ball rotates by about half a revolution during a bounce (as it does when spinning at 100 rev/s), then the deformation and energy loss might locally be reduced at all points in contact with the surface. However, there are no data available to support or refute this hypothesis. An alternative hypothesis is that sand on the court accumulates ahead of the ball to form a small ramp which deflects the ball to a greater height than one would normally expect. However, a ball served with heavy topspin can also bounce to around head height on Rebound Ace or DecoTurf (US Open) surfaces which are relatively smooth and hard and are not covered in sand.

There is ample scope for further measurements of e_x and e_y for a wide range of different balls and surfaces, at different speeds, angles, and spin values, that would help resolve issues such as the one just described. This type of measurement would make an interesting project for students and could even be done at home or on a relevant playing surface if the student has access to a digital video camera and a computer that accepts video input. Many ball impacts can be captured with sufficient detail even at 25 or 50 frames/s. The only other data on e_x that the author has seen concern unpublished measurements of ping-pong balls bouncing on different surfaces. As far as the author is aware, there are no published data on e_x for baseballs, basketballs, soccer balls, golf balls, cricket balls, handballs or any other balls commonly used in sport, either at low speed or at speeds of relevance to these sports.

X. CONCLUSIONS

The coefficient of restitution of a ball for a vertical bounce can be measured easily in terms of the bounce height or the bounce speed. A significantly greater effort is required to measure the horizontal coefficient of restitution, e_x, which partially explains why very little information is available on this parameter. Measurements and calculations for a tennis ball show that e_x is negative for an impact at low angles of incidence on a surface with a low coefficient of sliding friction, because the ball slides throughout the impact. At larger angles of incidence on surfaces with a high coefficient of sliding friction, e_x is typically close to zero, meaning that the collision is completely inelastic in the horizontal direction and that the ball begins to roll during the impact. However, e_x was observed to be about 0.2 for a low angle impact on surfaces with a high coefficient of sliding friction, indicating that the ball deforms elastically in the horizontal direction and that some of this energy is recovered on rebound. As a result, the ball spins faster and rebounds at a steeper angle than when $e_x = 0$. This behavior parallels that of a superball but the effect is typically much larger for a superball. A superball therefore spins faster than a tennis ball under the same oblique impact conditions.

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[8]The strings are designed to stretch in a direction perpendicular to the string plane, but they also move and stretch slightly in a direction parallel to the surface. Excessive movement of the strings parallel to the surface is forbidden by the rules of tennis because it allows a player to impart excessive spin to the ball. It is mainly for this reason that the cross strings must be woven alternately under and over the main strings. Strings that are not woven in this manner were available in the 1970's as "spaghetti" strings. These were quickly banned because they allowed the string plane to store elastic energy in a direction parallel to the surface, with the result that extra spin was imparted to the ball. Similarly, one could impart additional backspin to a golf ball or a baseball using a club or bat with a rubber surface. The ball would travel a greater distance, partly due to the additional lift force and partly because the impact would be softer, less energy would be dissipated in the ball and hence the speed of the ball off the club or bat would be greater.

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