A simple experiment for measuring the surface tension of soap solutions

F. L. Román, J. Faro, and S. Velasco
Departamento de Física Aplicada, Facultad de Ciencias, Universidad de Salamanca, 37008 Salamanca, Spain

(Received 2 August 2000; accepted 22 January 2001)

A simple experimental method for measuring the surface tension of a soap solution is proposed. In the experiment, a soap solution bubble is inflated by a syringe that is also connected to a precision manometer. By measuring the pressure change inside the bubble the surface tension can be calculated using the Young–Laplace equation. Experimental results for both toilet and dishwasher soap solutions are obtained.
[DOI: 10.1119/1.1365402]

I. INTRODUCTION

There are several techniques for making an accurate measurement of the surface tension of liquids.[1] Unfortunately, most of these techniques are not easy to implement as lab exercises because of the presence of contaminants in the liquid surface and the requirement of expensive apparatus, such as very sensitive and well-equilibrated balances for measuring the interfacial tension or accurate optical methods for measuring the shape of drops or bubbles. In this paper we propose a simple method for measuring the surface tension of a soap solution using a bubble method based on the Young–Laplace equation for the pressure difference between the inside and the outside of a bubble. The apparatus we report does not require especially careful handling and is suitable for general physics laboratories.

II. THEORY

The Young–Laplace equation for a spherical bubble is usually derived using mechanical arguments.[1,2] Here we present a derivation based on a thermodynamic approach that uses the minimization of the Helmholtz free energy function.[3] We consider that the bubble consists of an incompressible fluid film (soap solution) enclosing a given amount of gas (air) occupying a volume V_1. The Helmholtz function of the total system can be written as

$$F=F_{in}+F_{out}+F_{ss}+F_1+F_2, \tag{1}$$

where F_{in} and F_{out} are the free energies of the gas inside and outside the bubble, F_{ss} is the free energy of the volume of soap solution contained within inner and outer surfaces of area A_1 and A_2 and free energies F_1 and F_2, respectively. At constant temperature, the condition of minimum F,

$$dF_{in}+dF_{out}+dF_{ss}+dF_1+dF_2=0, \tag{2}$$

yields

$$-(P_{in}-P_{out})dV_1+\gamma dA_1+\gamma dA_2=0, \tag{3}$$

where P_{in} and P_{out} are the pressures inside and outside the bubble, respectively, γ is the surface tension, and we have taken $dF_{ss}=-P_{ss}dV_{ss}=0$ because the fluid is assumed to be incompressible. Finally, by considering the inner and outer surfaces as spherical with radii r_1 and r_2, respectively, and taking into account the fact that the incompressibility condition $dV_{ss}=0$ implies $r_1^2\,dr_1=r_2^2\,dr_2$, Eq. (3) leads to

$$\Delta P=P_{in}-P_{out}=2\gamma\left(\frac{1}{r_1}+\frac{1}{r_2}\right), \tag{4}$$

which is the well-known Young–Laplace equation for the pressure difference between the gas inside and outside of a spherical bubble. For large enough bubbles, the inner and outer radii are practically equal and Eq. (4) can be written as

$$\Delta P=\frac{4\gamma}{r}, \tag{5}$$

where r is the radius of the bubble.

III. EXPERIMENT

The experimental setup is shown schematically in Fig. 1. The apparatus consists of a 20 cm^3 plastic syringe connected both to a precision manometer and to a cylindrical tube ($\sim$10 cm length) with known inner and outer diameters. The tube is kept vertical with the help of a clamp. The precision manometer consists of an inclined tube of about 20 cm length connected to a reservoir containing a special tinted oil of known density. A graduated rule behind the inclined tube allows us to measure pressure differences up to 200 Pa with an uncertainty of ± 1 Pa. This kind of manometer is available commercially through several distributors,[4] but it can be also constructed without much difficulty.

After the system is prepared, we introduce the free end of the vertical tube into a soap solution. This produces a thick film of solution at this end. In order to eliminate the excess liquid (usually a pendant drop is formed at the end of the tube), we use a small piece of blotting paper. Next, we insufflate air into the tube by using the syringe (it is important that this insufflating process be performed very slowly). As a consequence, the soap solution film at the end of the tube begins to distend as the pressure inside the bubble is raised. The maximum pressure difference ΔP_{max} between the air pressure inside the bubble and atmospheric pressure is reached when the liquid film takes the form of a semispherical cap with radius equal to the radius of the tube. Since the

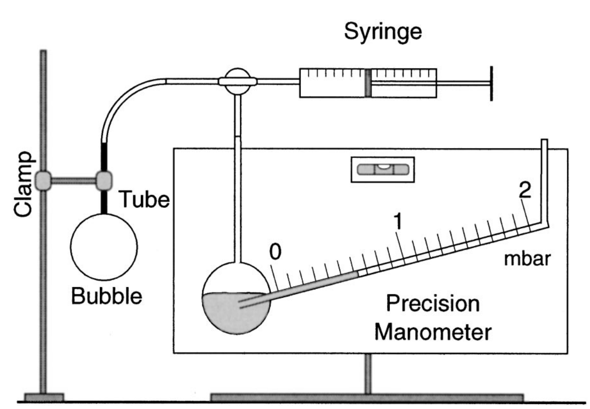

Fig. 1. Experimental setup for measuring the surface tension of a soap solution. The precision manometer measures the pressure difference between the inside and the outside of a bubble formed at the end of a tube by insufflating air with the syringe.

soap solution is not the same thickness as the tube wall, it is reasonable to use Eq. (5) with r given by an average radius $r_{\mathrm{ave}}=(r_{\mathrm{in}}+r_{\mathrm{out}})/2$, where r_{in} and r_{out} are the inner and outer radii of the tube, respectively. Once the maximum pressure difference is reached, if we continue the insufflating process (now more quickly) the diameter of the bubble increases while the pressure inside the bubble decreases. Finally, when the air contained in the syringe is totally insufflated one obtains a bubble with known radius, given by $r_{\mathrm{ref}}=(3V_0/4\pi)^{1/3}$, V_0 being the initial volume of the air in the syringe (in this case $V_0=20\,\mathrm{cm}^3$). The state of this bubble is taken as a reference state for which the pressure difference ΔP_{ref} is given by Eq. (5) with $r=r_{\mathrm{ref}}$. The selection of this state, instead of the initial one, as a reference state allows us to avoid pitfalls due to possible pressure changes in the initial state because of capillarity effects in the tube.

The difference between the maximum pressure difference and the pressure difference in the reference state is given by

$$\Delta P^* \equiv \Delta P_{\mathrm{max}} - \Delta P_{\mathrm{ref}} = \frac{4\gamma}{r^*}, \tag{6}$$

where $1/r^* \equiv 1/r_{\mathrm{ave}} - 1/r_{\mathrm{ref}}$. This procedure is repeated for several tubes with different diameters. Plotting ΔP^* vs $4/r^*$, a straight line should be obtained, where the slope is given by γ.

IV. RESULTS AND SUMMARY

We have obtained the surface tensions of two different aqueous soap solutions made from two different kinds of soap: one with a liquid dishwasher soap (soap A) and the other with a liquid toilet soap (soap B). Both soap solutions were made with a concentration of 10% by volume, and the experiment was done at constant temperature $T=19.5\pm0.1\,^\circ\mathrm{C}$ and atmospheric pressure $P_a=920\,\mathrm{mbar}$. We have used five metallic tubes obtained from an old telescoping radio antenna, each with different inner and outer diameters measured by a caliper to an uncertainty of ±0.005 cm. The initial volume in the syringe was $V_0=20\pm0.1\,\mathrm{cm}^3$, so that $r_{\mathrm{ref}}=1.684\pm0.003\,\mathrm{cm}$. It is worth noting that the final bubble volume is not exactly V_0 but $V_0-\Delta V$, where ΔV is the change in the volume above the manometer reservoir when the liquid level changes due to a pressure change. ΔV can be determined from the radius ($\sim$0.1 cm) of the inclined tube of the manometer and the difference between the lengths of the manometric liquid at the initial (before insufflating air) and final (after insufflating air) points. We find $\Delta V\sim0.02\,\mathrm{cm}^3$, which is negligible compared to V_0 when calculating r_{ref}. The experimental results are shown in Table I.

Table I. Experimental data values obtained using Eq. (6) for two soap solutions, A and B. The volume of total insufflated air was in both cases $V_0=20.0\pm0.1\,\mathrm{cm}^3$. The inner and outer radii of the tube where the bubble was formed are denoted as r_{in} and r_{out}.

r_{in} (cm)	r_{out} (cm)	ΔP^*_{A} (Pa)	ΔP^*_{B} (Pa)
0.160	0.180	47±1	55±1
0.200	0.220	37±1	42±1
0.225	0.245	33±1	38±1
0.280	0.300	25±1	30±1
0.385	0.405	17±1	20±1

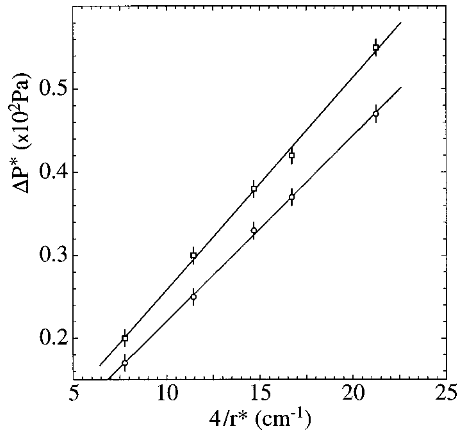

Fig. 2. Plot of ΔP^* vs $4/r^*$. Soap solution A (circles) and soap solution B (squares).

Values of ΔP^* vs $4/r^*$ are plotted in Fig. 2 for both soap solutions. Both sets of values of ΔP^* were fitted by linear regression. The values of γ obtained from the slope of each linear regression are, with their standard errors, $\gamma_A=0.0214\pm0.0018\,\mathrm{N/m}$ and $\gamma_B=0.0245\pm0.0018\,\mathrm{N/m}$. A typical value of γ reported in the pedagogical literature[5] for aqueous soap solutions is $\gamma=0.0250\,\mathrm{N/m}$.

In summary, a simple and accurate method for obtaining the surface tension of a soap solution is proposed that uses the Young–Laplace equation for spherical bubbles. The method consists in forming a bubble at the end of a tube with the help of a syringe and then measuring the pressure difference (of the air inside the bubble) between the maximum pressure state, corresponding to a bubble size with radius equal to the average of the inner and outer radii of the tube, and a reference state, corresponding to a bubble size with a volume equal to that of the air initially contained in the syringe.

ACKNOWLEDGMENT

This work was supported by the Comisión Interministerial de Ciencia y Tecnología (CICYT) of Spain under Grant No. PB 98-0261.

[1] A. W. Adamson, *Physical Chemistry of Surfaces* (Wiley-Interscience, New York, 1990), 5th ed., pp. 4–45.

[2] F. W. Sears and M. W. Zemansky, *Physics* (Addison–Wesley, Reading, MA, 1964), 3rd ed., Chap. 13.

[3] J. Pellicer, J. A. Manzanares, and S. Mafé, "The physical description of elementary surface phenomena: Thermodynamics versus mechanics," Am. J. Phys. **63**, 542–547 (1995).

[4] The precision liquid manometer used in this experiment is produced by PHYWE SYSTEME GMBH (D-37070 Göttingen, Germany), model 03091.00.

[5] Reference 2, Table 13.1.