Physics 404: Final Exam **Name (*print*):** ______________________________

"I pledge on my honor that I have not given or received any unauthorized assistance on this examination."

Dec. 18, 2012 ***Sign* Honor Pledge:**______________________________

1. Consider a mole of ideal gas, initially with V_i, p_i, T_i, 5 degrees of freedom (f=5), whose volume is doubled by various processes. ($V_f = 2\ V_i$). a) What is the *initial* internal energy U_i?

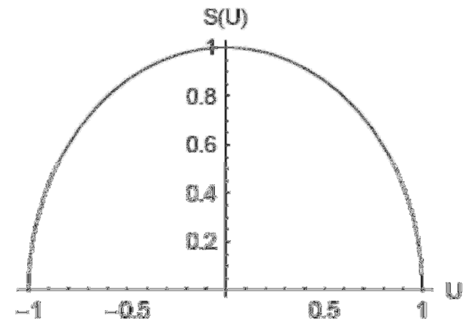
b) If the process is isothermal, find the final pressure p_f and the final energy U_f.

c) If the process is isobaric, find the final energy U_f.

d) If the process is adiabatic, find the final pressure p_f and the final energy U_f.

e) Suppose you constructed a closed cycle from the isobaric expansion (c) and an isothermal contraction (b), connected by an isochoric decrease in pressure at V_f. How much work is done in one cycle?

2. Suppose one can model a system by $S(U) = (1 - U^2)^{1/2}$, $1 \le U \le 1$, where S and U are dimensionless entropy and internal energy, respectively.



a) Find the temperature T as a function of U.

b) Draw a sketch of $T(U)$ under the $S(U)$ plot.

c) *Circle* those of the following that might have this $S(U)$:

2-state paramagnet   10-state system   rotating dimer   simple harmonic oscillator   Fermi gas

3. According to the equipartition theorem, what is the average [total] energy in equilibrium (at high enough temperature that all modes are excited) of

a) 4 atoms in a gas in 3D?

b) a (not vibrating) rigid dimer (diatomic gas molecule) in 2D?

c) a pair of balls, connected by a spring, moving in 1D

d) a solid of N atoms (in 3D, using the simplest model)

e) a vibrating, rotating dimer in 3D (with no freeze-out)?

4. In a magnetic field B we have $\mathrm{d}U = T\,\mathrm{d}S + m\,\mathrm{d}B$

Write the partial-derivative expressions for T and m, as well as the Maxwell relation between their derivatives. For each partial derivative, make clear which variable is held constant.

5. Consider a 3-state model with states having energies 0, ε, and 3ε. The middle state is doubly degenerate ($g_2 = 2$) while the other two are non-degenerate.

a) What is the partition function Z? (Don't try to combine terms.)

b) What is the free energy F?

c) What is the internal or average energy U?

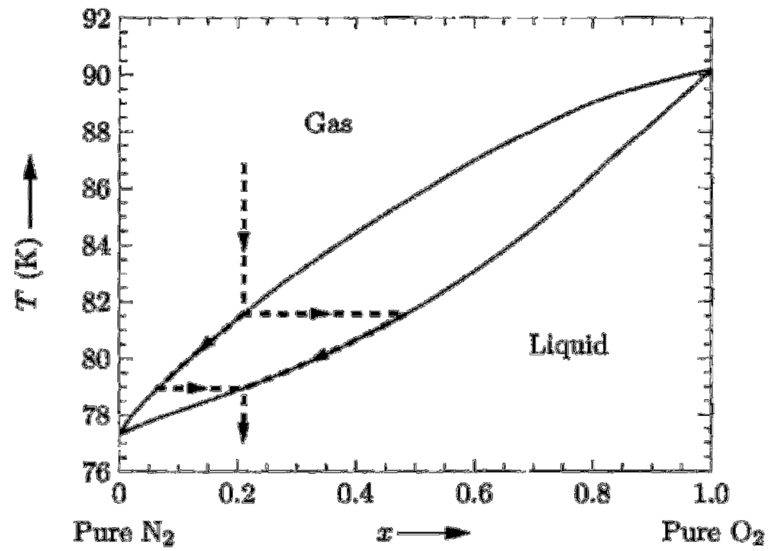
d) i) What is the heat capacity in the limit of low temperature T? <u>ii)</u> Is there evidence of an energy gap? (Explain briefly.)

e) What is the entropy S?

f) What is the probability that the system is in the state with energy 3ε?

6. For the Maxwell distribution in 3D, find the most probable speed (the mode).

7. The figure at the right is Schroeder's figure of the phase diagram of nitrogen and oxygen at atmospheric pressure. The dotted path shows the evolution of air as it is cooled. Suppose instead you start with a liquid with $x = 0.7$, i.e. 70% O_2, 30% N_2. a) Sketch similarly what happens when this liquid is heated. b) State clearly with the temperature T_1 at which the system starts boiling and T_2 at which it stops. c) For T slightly above T_1, describe fully the nature of the 2 phases that coexist.



8. In the models discussed in class, *x out* those of the following properties that depend on temperature T and *circle* those that do not. Draw a box around the one varying like T^4.

classical heat capacity of a solid　　entropy of a classical gas

Fermi energy　　chemical potential

power emitted from an ideal blackbody radiator　　density of states

9. All of the following, as dealt with in this course, assume non-interacting entities EXCEPT (circle one):

a) Electron gas　　b) Gas of bosons

c) Ideal gas　　d) [Paramagnetic] spins in a magnetic field

e) Phonons in the Einstein approximation　　f) Phonons in the Debye approximation

10. a) As in homework, find Z_1 of a relativistic gas ($\varepsilon = pc$) in 1D. (Reminder: let the lowest $\varepsilon \to 0$.

b) From Z_1 find U. (This result also follows from equipartition generalized to non-quadratic modes.)

11.a) Given $\mathcal{G}(\varepsilon) \propto \varepsilon^{(D/2)-1}$ show that the electron density of states in 1D is $\mathcal{G}_{1D}(\varepsilon) = A\, N\, \varepsilon^{-1/2} \varepsilon_F^{-1/2}$ and find A.

b) For N electrons (in 1D) at T=0, find $\langle\varepsilon\rangle$.

c) i) To find the lowest-order correction of $\langle\varepsilon\rangle$ for T>0 (in 1D), what general function $h(\varepsilon)$ should you plug into the Sommerfeld expansion? ii) What is the result? iii) What is the heat capacity?

12. a) Find the Helmholtz free energy of N simple harmonic oscillators (neglecting zero-point energy).

b) Is the chemical potential zero for this problem? Why (or why not)?

c) If the oscillators are fixed at constant density N/V, what is the pressure?

13. For Bose-Einstein condensation (in 3D), the key equation has the form $(N/V)\Lambda_T^3 = \mathrm{Li}_{3/2}(z)$.

a) What is the maximum value of the fugacity $z = e^{\beta\mu}$ in this problem? What would happen if z were larger?

b) Write the key equation in terms of the density of states and the Bose-Einstein distribution.

c) What would this equation be for a 2D system? (You do not need to derive your answer.) Why does it imply that there is no BEC in 2D?

d) For what value[s] of T/Tc are i) all the bosons in the ground state? ii) 7/8 of the bosons in the ground state iii) none (not "hardly any") of the bosons in the ground state?

e) On a graph, sketch chemical potential μ vs. T for both Bose and Fermi gases. Label clearly their values at $T = 0$, as well as the very approximate value of T where $\mu = 0$. Do the two curves ever cross?