Formulas for Final, PHYS404, Fall 2013 v1

$$\Omega(N,N_\uparrow)=\frac{N!}{N_\uparrow!\left(N-N_\uparrow\right)!}$$

$$\Omega(N,q)=\frac{(q+N-1)!}{q!\left(N-1\right)!}$$

$$\Omega(U,V,N)=f(N)V^N U^{fN/2}$$

$U=(f/2)\,Nk_BT$; determining f: 3 for atoms,
5 for diatomic molecules at low T, 7 at higher T,
2 for each direction of a[n Einstein] oscillator
Dimer rotational energy $= j\,(j+1)\,\varepsilon$

Ideal gas law $pV = Nk_BT = nRT$
van der Waals $pV = Nk_BT/(V\text{-}Nb)-aN^2/V^2$

Entropy $S = k_B \ln \Omega = \int dQ_{\text{rev}}/T$ $\quad 1/T = (\partial S/\partial U)_{V,N}$
Ideal gas $S = Nk_B\,[(V/(N\lambda_T^3) + 5/2]$; $\lambda_T = h/\sqrt{(2\pi m k_B T)}$
Paramag $S = Nk_B\,[\ln(2\cosh x) - x\tanh x]$; $x = \mu B/k_BT$
$U = -\mu B\tanh(\beta\mu B)$; $\quad M = N\mu\tanh(\beta\mu B)$

$\Delta U = Q + W_{\text{on}} = Q - W_{\text{by}}$ $\qquad \Delta S \geq 0$

Constants: $k_B \approx 10^{-4}$ eV/K $= 1.38\times10^{-23}$ J/K
C_V of 1 gm of water (ice) is 1 cal/K (~½ cal/K)
R ~ 8.3J/K $\qquad N_A = 6.02\times10^{23}$
Stirling: $\ln n! \approx n\ln n - n$

Expansions in $\varepsilon \ll 1$:
$\ln(1\pm\varepsilon) \approx \pm\varepsilon\,[-\varepsilon^2/2]$, $\quad \exp(\pm\varepsilon) \approx 1\pm\varepsilon\,[+\varepsilon^2/2!]$
$\Gamma(n+1)=n!$ (integer n); $\Gamma(1/2)=\sqrt{\pi}$; $\Gamma(x+1)=x\Gamma(x)$

Change in internal energy, change in temperature, heat, work, during "simple" processes:
isobaric ($\Delta p = 0$), isochoric ($\Delta V = 0 = W$), isothermal ($\Delta U = \Delta T = 0$), adiabatic ($Q = 0$).
Along an isobar, $W_{\text{by}} = p(V_f - V_i)$; along an isotherm $W_{\text{by}} = Nk_BT\ln(V_f/V_i)$
Along an adiabat pV^γ is constant, as is (using the ideal gas law) $TV^{\gamma-1}$ Note $\gamma = (f+2)/f$

Helmholtz and Gibbs free energies $\quad F(T,V,[N]) = U - TS + \mu N \qquad G(T,p,[N]) = U + pV - TS + \mu N$
$U(V,S\,[,N])\quad H(p,S\,[,N]) \qquad \Phi = U - TS - \mu N$

Thermo. identities: $dU = TdS - p\,dV + \mu dN$
$dF = -S\,dT - p\,dV + \mu\,dN$
$dG = -S\,dT + V\,dp + \mu\,dN$, etc.

Maxwell relations: 2nd deriv of thermo functions do not depend on order of derivs

$\Delta S_{\text{mix}} = -Nk_B[x\ln x + (1-x)\ln(1-x)]$

Expansion $\beta = V^{-1}\,\partial V/\partial T|_p \qquad \kappa_T = -V^{-1}\,\partial V/\partial p|_T$
$S = \int C/T\,dT$
$F = -k_BT\ln Z \qquad U = -\partial\ln Z/\partial\beta$
$Z_N = Z_1^N/N!$ for indistinguishable $\quad \mu = (\partial F/\partial N)_{T,V}$

Non-relativistic gas $Z_1 = (V/\lambda_T^3)\,Z_{\text{int}}$

$Z = \Sigma_i\, g_i\, e^{-\beta\varepsilon i} \qquad \mathcal{Z} = \Sigma_i\, g_i\, e^{-\beta(\varepsilon i - \mu Ni)} = \Sigma_N z^N Z_N$
[NB εi means ε_i]

$Z_{\text{rot}} \approx k_BT/2\varepsilon$ for $k_BT \gg \varepsilon$, dimer of identical atoms

Maxwell 3D $\quad D(v)=\left(\frac{m}{2\pi k_BT}\right)^{3/2} 4\pi v^2 \exp\left(-\frac{mv^2}{2k_BT}\right)$

Ideal gas $\Phi = -k_BT\ln\mathcal{Z} = -pV$

$\mathcal{Z}_{FD/BE} = (1 \pm \exp[-\beta(\varepsilon-\mu)])^{\pm1} \qquad \beta = 1/k_BT$

$$\bar{n}_{\frac{FD}{BE}}(\varepsilon;T,\mu)=\frac{1}{\exp[\beta(\varepsilon-\mu)]\pm1}=\frac{1}{z^{-1}\exp(\beta\varepsilon)\pm1}$$

$\varepsilon \propto n^p$: $\quad \varepsilon=\frac{h^2}{8mL^2}\left(n_x^2+n_y^2+n_z^2\right)=\frac{h^2n^2}{8mL^2}$ or $\varepsilon=\frac{hcn}{2L}$

Fermi energy $E_F = (h^2/8m)(3N_{el}/\pi V)^{2/3}$

$\mathcal{G}_{FD}(\varepsilon) \propto (\varepsilon/\varepsilon_F)^{(D/2)-1}/\varepsilon_F$; $\mathcal{G}(\varepsilon) \propto \varepsilon^{(D/p)-1}$

$$\int^{\infty} h(\varepsilon)\bar{n}_{FD}(\varepsilon;T)\,d\varepsilon=\int_0^{\varepsilon_F} h(\varepsilon)\,d\varepsilon+\frac{\pi^2}{6}\left(k_BT\right)^2\left[h'(\varepsilon_F)-h(\varepsilon_F)\frac{G'(\varepsilon_F)}{G(\varepsilon_F)}\right]$$
[NB: G is $\mathcal{G}$]

Photons $u(\varepsilon) = [8\pi\varepsilon^3/(hc)^3]\;\bar{n}_{BE}\left(\varepsilon;T,0\right) \qquad \varepsilon = hf$

$U/V = (8\pi^5/15)[(k_BT)^4/(hc)^3]$

Wien: $\lambda_{\max}T = 0.0029$ m K; $\nu_{\max} \neq c/\lambda_{\max}$

Power from perfect blackbody radiator: $\sigma\times\text{area}\times T^4$

Debye $\quad T_D = (hc_s/2k_B)(6N/\pi V)^{1/3}$

$$U=\frac{9Nk_BT^4}{T_D^3}\int_0^{T_D/T}\frac{x^3}{e^x-1}\,dx$$

BEC $\quad \Gamma(\nu)\,Li_\nu=\int_0^\infty\frac{x^{\nu-1}dx}{z^{-1}e^x-1} \qquad N = (V/\lambda_T^3)Li_{3/2}(z)$

$U = (3/2)\,k_BT\,Li_{5/2}(z) \qquad N/V = 2.612/(\lambda_T(T=T_c))^3$

$E_{\text{Ising}} = -J\,\Sigma_{<i,j>}\,s_i\,s_j \qquad$ Mean field $\bar{s} = \tanh(\beta qJ\,\bar{s})$