

Physics 404: Midterm Exam

Name (*print*): _____

"I pledge on my honor that I have not given or received any unauthorized assistance on this examination."

Oct. 18, 2012 **Sign Honor Pledge:** _____

Don't get bogged down on any problem you find difficult. Skip ahead and go through all 4 pages, then come back to it afterwards!!

1. An ideal gas of N atoms (e.g. He) has an initial pressure p_0 and an initial volume V_0 .

i) What is the initial temperature T_0 ?

Suppose the volume triples (in a quasistatic, reversible fashion).

a) If the tripling is done isothermally i) what is the final pressure?

ii) what is the final temperature?

iii) how much work is done *on* the gas?

iv) how much heat flows into the gas?

b) If the tripling is done isobarically, i) what is the final pressure?

ii) what is the final temperature?

iii) how much work is done *on* the gas?

iv) how much heat flows into the gas?

c) If the tripling is done adiabatically, i) what is the final pressure? ii) What is the explicit value of γ ?

iii) How much work does the gas do? (What is W_{by} ?) iv) What is the corresponding ΔU ?

d) What is the ratio of the heat flowing into the gas in the isobaric case and the isothermal case?
(Use $\ln 3 \approx 1.1 \approx 1$)

2. a) Using Stirling's approximation, work out the entropy of the N -spin paramagnet in the macrostate in which $\frac{1}{4}$ of the spins are in one direction (say up) and $\frac{3}{4}$ in the opposite direction (down), simplifying as much as possible and showing that the answer is proportional to Nk_B .

b) How does this entropy compare with that when $\frac{3}{4}$ of the spins are up and $\frac{1}{4}$ are down?

c) How do you expect your answer to compare with the entropy of such a system in this approximation with half the spins in each direction?

3. Consider the nearly empty spreadsheet for finding the dimensionless heat capacity of an Einstein solid with $N = 100$ oscillators. For line 6 ($q=3$, $U= 3\epsilon$, i.e. 3 eps), write below the formulas that you would insert in each column. You can use either normal math expressions or Excel functions. Refer to the entries that would be in the various boxes in the usual spreadsheet way. (Thus, A6 has value 3.)

	A	B	C	D	E
1	N=	100			
2	q	Omega	S/k	kT/eps	C/Nk
3	0				
4	1				
5	2				
6	3				
7	4				
8	5				
9	6				
10	7				
11	8				
12	9				

B6:

C6:

D6:

E6:

What value does C/Nk (i.e. C/Nk_B) approach as temperature T gets very small?

What value does C/Nk (i.e. C/Nk_B) approach as temperature T gets very large (presuming that N is not so large that the program breaks down).

C/Nk reaches about half its high- T saturation value when $k_B T/\epsilon$ is about (circle correct answer)

1/10

1/3

1

3

10

4. Cryophorus-inspired problem: Assume for simplicity that the [molar] latent heat [of boiling] L and the molar specific heat c of a liquid (say water) are independent of temperature T .

a) If a fraction x of the liquid evaporates, what is the temperature decrease ΔT of the remaining liquid?

b) What limits the magnitude of ΔT ?

c) How in fact does L change with decreasing temperature?

Increases or decreases? By a lot, a little, or a negligible amount?

5. According to the equipartition theorem, what is the average [total] energy (in the classical regime) of

a) a solid of N atoms (in 3D, using the simplest model)

b) a vibrating dimer in 3D

c) N atoms in a gas in 2D (confined to move in a plane)?

d) 2 rigid dimers (not vibrating) in 2D?

6. For the grand potential (or free energy) we have the relation: $d\Phi = -S dT - p dV - N d\mu$

a) Write the resulting partial-derivative formulas for S and N , being careful to specify what is held constant.

b) Write the Maxwell relation relating derivatives of S and N .

Study Guide for Midterm, PHYS404, Fall 2012

ver. 1.0

Heat, work (on or by), temperature, [internal] energy, heat capacity, latent heat, entropy, enthalpy

3 basic models: paramagnet (2-state), Einstein solid, ideal gas

– What N and q (or U) mean for each, and the resulting multiplicities $\Omega(N, \dots)$

$$\Omega(N, N_{\uparrow}) = \frac{N!}{N_{\uparrow}!(N - N_{\uparrow})!}$$

$$\Omega(N, q) = \frac{(q + N - 1)!}{q!(N - 1)!}$$

$$\Omega(U, V, N) = f(N)V^N U^{fN/2}$$

2 weakly interacting systems, thermodynamic limit
Application of models to other physical systems

Equipartition theorem at thermal equilibrium:

$U = (f/2) Nk_B T$; determining f : 3 for atoms,
5 for diatomic molecules at low T , 7 at higher T ,
2 for each direction of an Einstein oscillator

Ideal gas law $pV = Nk_B T = nRT$

Entropy $S = k_B \ln \Omega = \int dQ_{\text{rev}}/T$

Ideal gas $S = Nk_B [(V/(N\Lambda^3)) + 5/2]$; $\Lambda = h/\sqrt{2\pi mk_B T}$

Paramag $S = Nk_B [\ln(2 \cosh x) - x \tanh x]$; $x = \mu B/k_B T$

Quasistatic vs. free expansion, microstate vs. macrostate; intensive vs. extensive

Laws of thermodynamics, and what they mean

$$\Delta U = Q + W_{\text{on}} = Q - W_{\text{by}} \quad \Delta S \geq 0$$

Spreadsheet computations of Ω , S , U , T , C for paramagnets and Einstein solid

Constants: $k_B \approx 10^{-4}$ eV/K = 1.38×10^{-23} J/K

C_V of 1 gm of water (ice) is 1 cal/K ($\sim 1/2$ cal/K)

$R \sim 8.3$ J/K $N_A = 6.02 \times 10^{23}$

Meaning of mole: mass = atomic wgt grams

Heat engines, Carnot cycles, refrigerators, efficiency, coefficient of performance

Very large numbers; Stirling's approximation,

$\ln n! \approx n \ln n - n$, and how to use it

Expansions in $\epsilon \ll 1$:

$$\ln(1 \pm \epsilon) \approx \pm \epsilon [-\epsilon^2/2], \quad \exp(\pm \epsilon) \approx 1 \pm \epsilon [+ \epsilon^2/2!]$$

pV diagrams, inc. Otto and Diesel cycles

Change in internal energy, change in temperature, heat, work, during "simple" processes:

isobaric ($\Delta p = 0$), isochoric ($\Delta V = 0 = W$), isothermal ($\Delta U = \Delta T = 0$), adiabatic ($Q = 0$).

Along an isobar, $W_{\text{by}} = p(V_f - V_i)$; along an isotherm $W_{\text{by}} = Nk_B T \ln(V_f/V_i)$

Along an adiabat pV^γ is constant, as is (using the ideal gas law) $TV^{\gamma-1}$ Note $\gamma = (f+2)/f$

Helmholtz and Gibbs free energies $F(T, V, [N]) = U$

$-TS + \mu N$ $G(T, p, [N]) = U + pV - TS + \mu N$

$U(V, S, [\mu])$ $H(p, S, [\mu])$ $\Phi = U - TS - \mu N$

Electrolysis, fuel cells

Thermo. identities: $dU = TdS - p dV + \mu dN$

$$dF = -S dT - p dV + \mu dN$$

$dG = -S dT + V dp + \mu dN$, etc. & uses

Maxwell relations: 2nd deriv of thermo functions do not depend on order of derivs

$$\Delta S_{\text{mix}} = -Nk_B [x \ln x + (1-x) \ln(1-x)]$$

$$\beta = V^{-1} \partial V / \partial T|_p \quad \kappa_T = -V^{-1} \partial V / \partial p|_T$$