**Physics 404: Second Midterm Test   Name (*print*):** ___________________________________

*"I pledge on my honor that I have not given or received any unauthorized assistance on this examination."*

April 17, 2008   ***Sign* Honor Pledge:**_________________________________________

Don't get bogged down on any problem you find difficult. Skip ahead and go through all 4 pages, then come back to it afterwards!! If you find a question ambiguous, ask or write a short explanation of why.) Much of the reference information at the end should be of help.

1. A block of lead with heat capacity C is cooled from $T_h$ by plunging it into a cold bath at $T_\ell$.
a) What is the change in entropy of the lead?

b) What is the change in entropy of the water in the cold bath?

c) What is the total change in entropy? Is it positive, negative, or 0?

d) If the lead block is instead cooled in steps by plunging it successively into baths at intermediate temperatures, indicate how the various entropies above change.

2. Consider expansion of a thermally isolated ideal gas, with initial pressure $p_i$, to 3 times the initial volume.
a) If the expansion is done reversibly (e.g. by pushing on a piston), how much work is done *by* the gas?

b) What is the change in entropy of the gas $\Delta S_{gas}$?

c) What is $\Delta S_{gas}$ if the expansion is done irreversibly as in a free or Joule expansion?

d) What is the *total* change in entropy $\Delta S_{universe}$ if i) the expansion is reversible?

and ii) if the expansion is free (Joule)?

3. a) A system has 3 states which occur with probability ½, ¼, and ¼. What is its entropy? (Write your answer as a multiple of ln 2.)

b) If the energies of these states are 0, 1, and 2 eV, respectively, what are the internal energy U and the Helmholtz free energy F? Take $k_B T = 1/(10 \ln 2)$ eV.

4. Below are many formal expressions used in the chapters covered by this test, and possibly helpful here as well. Identify each from the following list (not all of which are needed for the matching):

$\alpha_f$ $\beta_P$ $\beta_S$ [expansivities] $C_P$ $\gamma$ G H $\kappa_T$ $\kappa_S$ [compressibilities] p S dS

_____ a) $U + pV$

_____ b) $T(\partial S/\partial T)_P$

_____ c) $đQ_{rev}/T$

_____ d) $(1/L)\,(\partial L/\partial T)_f$

_____ e) $-(1/V)\,(\partial V/\partial p)_T$

_____ f) $k_B \ln \Omega$

_____ g) $(1/V)\,(\partial V/\partial T)_P$

_____ h) $C_P/C_V$

_____ i) $-(\partial F/\partial V)_T$

5. For an elastic rod, length L and tension *f*, find $(\partial f/\partial L)_T(\partial L/\partial T)_f$ Indicate what it tells you, physically.

6. Does the linear expansivity (at constant tension *f*) of a material have to be positive? If yes, why? If no, give an example for which it is negative.

7. a) Show that $(\partial V/\partial T)_S = C_V/[T(\partial p/\partial T)_V]$ and b) use it to find the isentropic expansivity of an ideal gas.

8. For a magnetic system, U = U(S,m); derive the Maxwell relation based on dU. (See reference info.) For half credit, you may instead derive the relation based on dU for our standard gas (p-V) system.

9. At temperature *T*, a dimer moves in 3 dimensions; the distance between the two atoms is rigidly fixed.
a) How many degrees of freedom *f* does the dimer have?

b) What is its mean energy U?

c) What is its heat capacity C?

d) If, instead, the dimer is constrained so that both atoms are always some fixed distance from a smooth surface. (Imagine it as sliding along an icy surface, moving in 2D.)
What is i) its mean energy U and ii) its heat capacity?

e) Suppose now that the distance between the dimer is not a constant but can vibrate. However, rather than the usual harmonic spring potential, the potential around the equilibrium separation $r_0$ has the form $\alpha(r - r_0)^6$. How does this unusual vibration *change* the mean energy?

10. Consider a model system with partition function $Z = (k_B T/\varepsilon)^2 = 1/(\beta\varepsilon)^2$.

a) What is the free energy F?

b) What is the internal energy U?

c) What is the entropy S?

d) What is the heat capacity C?

11. System 1 has partition function $Z_1$ and energy $U_1$. If system 2 has partition function $Z_2 = e^{-\beta\varepsilon} Z_1$, how is $U_2$ related to $U_1$?

12. What is the partition function of a system of 100 energy levels, spaced by $\varepsilon$, i.e., with energies $0, \varepsilon, 2\varepsilon, 3\varepsilon, \ldots, 99\varepsilon$?

13. For each statement, write U, S, C, or F (or possibly several of them) if the statement is always true (at least for the models studied) for internal energy, entropy, heat capacity, or Helmholtz free energy:

a) Goes to 0 as $T \to 0$.

b) Has a magnitude that increases monotonically (i.e. never decreases) with increasing T

c) Approaches a constant for large T.

d) Is always non-negative

e) Is minimized in systems with fixed T and V.

f) Is minimized in systems with fixed S and V.

*Some reference information:*

$R = 8.3$ J mole$^{-1}$ K$^{-1}$

$\Delta S = -Nk_B[x \ln x + (1-x) \ln(1-x)]$

$dU = TdS - pdV = TdS + fdL = TdS + Bdm$

$dH = TdS + Vdp$

$dF = -SdT - pdV$

$dG = -SdT + Vdp$

$E_T = (\sigma/\varepsilon) = (L/A)(\partial f/\partial L)_T$

$\chi = \lim_{H\to 0} M/H \approx \mu_0 M/B$

$\Sigma_{n=0}^{N}\ x^n = (1 - x^{N+1})/(1 \;\; x)$