Physics 404: Final Exam **Name (*print*): ________________________________**

"I pledge on my honor that I have not given or received any unauthorized assistance on this examination."

May 20, 2008 ***Sign* Honor Pledge:____**

Don't get bogged down on any problem you find difficult. Skip ahead and go through all 4 pages, then come back to it afterwards!! If you find a question ambiguous, ask or write a short explanation of why.) Much of the reference information at the end should be of help.

1. a) Show that the peak of u_ω (vs. ω) for blackbody radiation satisfies $e^x(n-x) = n$, where $x \equiv \hbar\omega/k_BT$; be sure to identify the value of n.

b) What is the ratio of the power radiated by a sphere at T_2 compared to that of another at T_1, assuming all else is the same for the two spheres?

d) If the spheres have different radii R_1 and R_2, for what ratio R_2/R_1 is the power radiated from the two spheres the same? (Or will the power ratio remain the same as in (b)?)

2. A 2-state system has energies $E_0 = 0$ and $E_1 > 0$. a) What is the ratio of occupations N_1/N_0?

b) What is the mean energy U?

c) What is the free energy F?

d) For a process described by the Einstein A coefficient, is a photon emitted or absorbed, and what is its energy?

3. a) What is the entropy associated with putting a red, a green, and a blue ball into 5 bowls? (no more than one ball per bowl)?

b) What is this entropy if the balls are the same color (indistinguishable)?

4. Consider the chemical reaction $2CO + O_2 \leftrightarrow 2CO_2$

a) What is the relationship (in equilibrium) between the chemical potentials of the 3 gases?

b) What is the equilibrium constant *K* in terms of the partial pressures of the 3 gases?

c) If the reaction is exothermic, what happens to the relative concentration of CO_2 if the temperature is raised?

5. a) Recall (hopefully) that from dimensional arguments $g(E) = A\,N\,E^{1/2}/E_F^{3/2}$ for fermions in 3D. Find the constant A.

b) What is the value of E^2 for Fermi gas at T=0? Express your answer in terms of N and E_F.

c) What is the leading correction in T at finite temperature?

6. For Bose-Einstein condensation, the key equation has the form $(N/V)\,\lambda_{th}^3 = Li_{3/2}(z)$.

a) What is the maximum value of $z = e^{\beta\mu}$ in this problem?

b) Write the condition for T_c, where the condensate begins to form.

c) What fraction of the gas is in the ground state i) at T=0? ii) at T= ½ T_c? iii) at T= 2 T_c?

7. a) Write N and S in terms of partial derivatives of the grand potential Φ_G (recall $d\Phi_G = -SdT - pdV - Nd\mu$). For the partials, make clear what is held constant.

b) Using the above, find a Maxwell relation between N and S. (Again, for the partials, make clear what is held constant.)

8. Consider two particles, each of mass m, connected by a spring, moving in one dimension, along a line, one ahead of the other. The particles undergo simple harmonic motion (vibrate) with angular frequency ω, and the center of mass moves with velocity **v**.

a) What is the partition function Z_{vibr} associated with the vibration?

b) What is the partition function Z_{trans} for the center-of-mass motion?

c) What is the full partition function (in terms of Z_{vib} and Z_{trans}) for the two particles, assuming they are distinguishable.

d) How does your answer to c) change if they are indistinguishable?

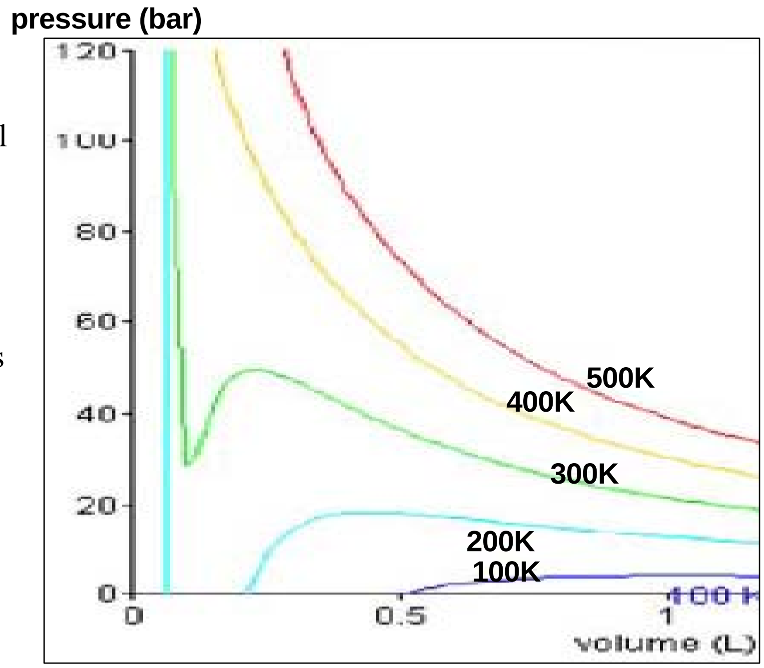
e) According to equipartition, what is the mean energy of the system?

9. a) For the middle isotherm, at T=300K, of the depicted *mole* of van der Waals gas, indicate by *hashing/thickening the curve* where the isothermal compressibility is unphysical.

b) Draw the Maxwell construction for this isotherm.

c) From the endpoint of this Maxwell line, what is the density (in moles/liter) of the liquid phase in the coexistence regime at this T?

d) Estimate (to within ±25K) the critical temperature.



10. a) For what speed v is the Maxwell-Boltzmann distribution maximum? I.e. what is the mode speed?

b) In calculating the mean kinetic energy of a classical gas described by the Maxwell-Boltzmann distribution, at what speed is the contribution maximum?

11. For each of the following properties of classical gases, answer A) is raised by raising temperature T, B) is rather unchanged by raising T, C) decreases when T is raised, D) can't tell for sure.:

a) Pressure

b) Ratio of effusion flux (through a small hole) to pressure

c) Mean free path

d) Mean collision time

e) Compressibility κ_T [assuming the gas is ideal]

12. Consider various processes that double the volume of a system.

What is the change in energy U of an ideal gas if the doubling of V occurs isobarically at pressure p?

What is the work done *by* an ideal gas if the doubling of V occurs isobarically at pressure p?

What is the work done *by* an ideal gas if the doubling of V occurs isothermally at T?

What is the work done *by* a *van der Waals* gas if the doubling of V occurs isothermally at $T > T_C$?

13. a) If the latent heat L and the volume difference ΔV (volume per mole of the higher-temperature phase minus volume per mole of the lower-temperature phase) are both positive and are both independent of T, show that the phase boundary has negative curvature d^2p/dT^2.

b) Which of these 4 assumptions is most obviously violated for the water-ice transition?

14. a) On what very general basis do we know that the heat capacity must go to 0 as $T \to 0$?

b) How does the heat capacity at low T reveal whether there is a gap (finite energy) between the ground state and the first excited state?

c) For each of the following, indicate the T dependence of the heat capacity at very low temperature. For either power-law or exponential dependence, you do not need to include any prefactor, just T raised to the appropriate power or e to some exponent (which you do have to give explicitly!).

_______ 2-level system or paramagnet

_______ Liquid ^{4}He or other degenerate Bose gas

_______ Electrons in a metal or other degenerate Fermi gas (in 3D)

_______ Simple harmonic oscillator

_______ Lattice vibrations, Einstein model

_______ Lattice vibrations, Debye model (in 3D)

_______ Lattice vibrations, Debye model, **in 2D**

Some reference information:

$\gamma = (2/f) + 1$ $\qquad$ $U = (f/2)\, n_m RT$

molecules hitting unit area of wall in unit time with speeds between v and v + dv and travelling at angles between θ and θ + dθ:

$v \cos\theta\, n\, f(v)\, dv\, \frac{1}{2} \sin\theta\, d\theta$

flux of molecules $\Phi = \frac{1}{4} n \langle v \rangle = p/(2\pi m k_B T)^{1/2}$

mean scattering time $\tau = 1/(n\,\sigma\,\langle v_{relative}\rangle)$ $\qquad$ mean free path $\lambda \approx 1/[\sqrt{(2)}\, n\, \sigma]$

$\eta = \frac{1}{3} n\, m\, \lambda \langle v \rangle$ $\qquad$ $\kappa = \frac{1}{3} C_V \lambda \langle v \rangle$ $\qquad$ $D = \frac{1}{3} \lambda \langle v \rangle$

nucleon (proton or neutron) mass: 1.7×10^{-27} kg.

$R = 8.3$ J mole^{-1} K^{-1}

$\Delta S = -Nk_B[x \ln x + (1-x) \ln(1-x)]$

$dU = TdS - pdV = TdS + fdL = TdS + Bdm$

$dH = TdS + Vdp$ $\qquad$ $dF = -SdT - pdV$ $\qquad$ $dG = -SdT + Vdp$

$E_T = (\sigma/\varepsilon) = (L/A)(\partial f/\partial L)_T$

$\chi = \lim_{H\to 0} M/H \approx \mu_0 M/B$

$\Sigma_{n=0}^{N}\ x^n = (1 - x^{N+1})/(1\ \ x)$

$\Gamma(n+1) = n!$ (for integer n) ; $\quad \Gamma(1/2) = \sqrt{\pi}$; $\quad \Gamma(x+1) = x\,\Gamma(x)$

$dp/dT = L/(T\,\Delta V)$

$d\Phi_G = -SdT - pdV - Nd\mu$

$p = RT/(V-b) - a/V^2$

$\lambda_{th} = h/(2\pi m k_B T)^{1/2}$

$\tilde{g}(k)dk \sim L^d k^{d-1}\, dk$ so $g(E)dE = \tilde{g}(k(E))\,(dk/dE)\, dE$; $\; E \propto k^2$ (massive) or k (pho(t/n)ons)

$$\int^{\infty} \phi(E)\, f_{FD}(E) dE = \int_0^{E_F} \phi(E) dE + \frac{\pi^2}{6}\left(k_B T\right)^2 \left[\phi'(E_F) - \phi(E_F)\frac{g'(E_F)}{g(E_F)}\right]$$

$$\int_0^{\infty} \frac{E^{n-1} dE}{z^{-1} e^{\beta E} - 1} = (k_B T)^n\, \Gamma(n)\, Li_n(z)$$