Physics 404: First Midterm Test **Name (*print*): ___________________________________**

"I pledge on my honor that I have not given or received any unauthorized assistance on this examination."

March 11, 2008 ***Sign* Honor Pledge:____**

Don't get bogged down on any problem you find difficult. Skip ahead and go through all 4 pages, then come back to it afterwards!! If you find a question ambiguous, ask the TA and/or write a short explanation of why.)

1. Short answer or multiple choice (in which case, circle the correct answer):

a) What is the probability that a particle has not scattered by time 2τ?

0 1/2 1/4 e^{-1} e^{-2} e^{-4} < 0.00001

b) What is $k_B T$ (to one significant digit) for T = 100 K? Indicate clearly which unit of energy you are using!

c)) Rewrite the thermal conductivity κ in terms of τ rather than λ.

2. Suppose a system has 3 possible energies: 0, ε, and 2ε.
a) Write the general expression for $\langle E \rangle$ in this case. (No derivation needed.)
b) Evaluate $\langle \beta E \rangle$ if $\exp(-\beta\varepsilon) = 0.1$, or equivalently $\beta\varepsilon = 2.3$.

3. Using Stirling's approximation, show that ln [N!/(N/2)!] can be written as (N/2) times some expression, and find that multi-term expression.

4. Show that $\ln \Omega(E-\varepsilon) \approx \ln \Omega(E) - \varepsilon/k_B T$ for $\varepsilon \ll E$. (Reminder: first do a Taylor expansion of $\ln \Omega$ to first order.)

5. a) Note that 1/3 appears in the pressure ($p = \frac{1}{3} n m \langle v^2 \rangle$), viscosity η, the thermal conductivity κ, and the diffusion constant D.
For which cases does it come from the integral

$$\frac{1}{2}\int_0^{\pi} \cos^2\theta \sin\theta d\theta \quad ?$$

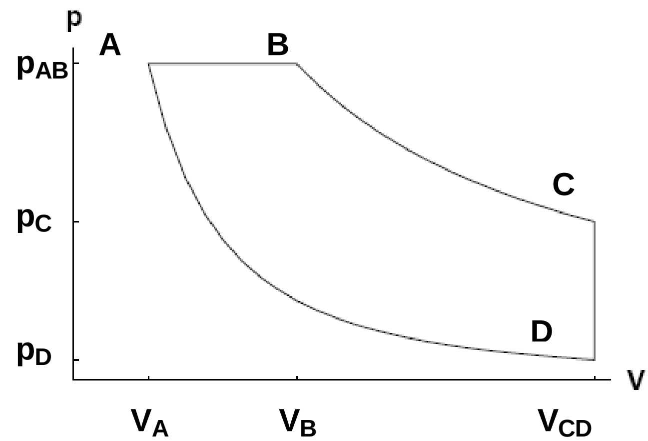
b) For **2** of the 3 transport coefficients η, κ, and D (indicating clearly which you are picking), state what is being transported.

6. a) Find $\langle 1/v^2 \rangle$ for a Maxwell-Boltzmann gas in equilibrium at temperature T. (For partial credit, you may instead find $\langle v \rangle$, as in class and the homework problem, for parts a and b.)

b) Find the same quantity for effusion. Just quickly indicate how the steps in the previous part change; no need to rewrite all the steps.

c) Find (don't just write down) the most probable value of the speed for effusion of this gas.

7. Consider the sketched the pV diagram of a cycle of an ideal gas, using all 4 of the processes we have studied. Make the usual assumptions about reversibility and quasistatic motion along the paths.



a) For each, indicate the name of the process and what is constant along it. Note that AD and BC are not the same process; AD is steeper than BC.

AB

BC

CD

DA

b) Draw arrows along the paths to show the direction followed if the cycle represents a heat engine that does work on the environment.

c) Along which legs does $\Delta U = (f/2)\, N\, k_B\, (T_f - T_i)$? [Ti and T_f are the initial and final temperatures, respectively, and could be T_A, T_B, T_C or T_D.]

d) Along which leg does the incoming heat Q (or ΔQ) $= Nk_BT \ln(V_f/V_i)$?

e) What are the highest and the lowest temperatures, T_h and T_ℓ, around the cycle? (Respond using T_A, T_B, T_C or T_D.)

f) The efficiency of the heat engine is

0 less than $1 - (T_\ell/T_h)$ equal to $1 - (T_\ell/T_h)$ greater than $1 - (T_\ell/T_h)$ at least 1

g) Along which path does Q (or ΔQ) $= C_V\, (T_f - T_i)$?

h) Along which path[s] does $pV = Nk_BT$?

i) Along which path[s] does $T\, V^{\gamma-1} = \text{const.}$?

Some reference information:

$$\int_0^\infty x^n e^{-x} dx = n!$$

$$\int_0^\infty x^{2n+1} e^{-x^2} dx = \frac{n!}{2}$$

$$\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$$

$$\int_0^\infty x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{4}$$

$$\int_0^\infty x^4 e^{-x^2} dx = \frac{3\sqrt{\pi}}{8}$$

$$\int_0^\infty x^6 e^{-x^2} dx = \frac{15\sqrt{\pi}}{16}$$

$\gamma = (f/2) + 1$ $\quad$ $U = (f/2)\, n_m RT$

Requested equations:

molecules hitting unit area of wall in unit time with speeds between v and v + dv and travelling at angles between θ and θ + dθ:

$v \cos\theta \; n \; f(v) \; dv \; \frac{1}{2} \sin\theta \; d\theta$

flux of molecules $\Phi = \frac{1}{4} n \langle v \rangle = p/\sqrt{(2\pi m k_B T)}$

mean scattering time $\tau = 1/(n\, \sigma \langle v_{relative} \rangle)$

mean free path $\lambda \approx 1/[\sqrt{(2)}\, n\, \sigma]$

$\eta = \frac{1}{3} n\, m\, \lambda \langle v \rangle$ $\quad$ $\kappa = \frac{1}{3} C_V \lambda \langle v \rangle$ $\quad$ $D = \frac{1}{3} \lambda \langle v \rangle$

nucleon (proton or neutron) mass: 1.7×10^{-27} kg.

$R = 8.3$ J mole^{-1} K^{-1}